Question 1

The fractional quantum Hall effect (FQHE) was discovered by D. C. Tsui and H. Stormer at Bell Labs in 1981. In the experiment electrons were confined in two dimensions on the GaAs side by the interface potential of a GaAs/AlGaAs heterojunction fabricated by A. C. Gossard (here we neglect the thickness of the two-dimensional electron layer). A strong uniform magnetic field B was applied perpendicular to the two-dimensional electron system. As illustrated in Figure 1, when a current I was passing through the sample, the voltage V_{H} across the current path exhibited an unexpected quantized plateau (corresponding to a Hall resistance $R_{\mathrm{H}} = 3h/e^2$) at sufficiently low temperatures. The appearance of the plateau would imply the presence of fractionally charged quasiparticles in the system, which we analyze below. For simplicity, we neglect the scattering of the electrons by random potential, as well as the electron spin.

(a) In a classical model, two-dimensional electrons behave like charged billiard balls on a table. In the GaAs/AlGaAs sample, however, the mass of the electrons is reduced to an effective mass m^* due to their interaction with ions.
(i) (**2 point**) Write down the equation of motion of an electron in perpendicular electric field $\vec{E} = -E_y\hat{y}$ and magnetic field $\vec{B} = B\hat{z}$.
(ii) (**1 point**) Determine the velocity v_{s} of the electrons in the stationary case.
(iii) (**1 point**) Which direction is the velocity pointing at?

(b) (**2 points**) The Hall resistance is defined as $R_{\mathrm{H}} = V_{\mathrm{H}}/I$. In the classical model, find R_{H} as a function of the number of the electrons N and the magnetic flux $\phi = BA = BWL$, where A is the area of the sample, and W and L the effective width and length of the sample, respectively.

(c) (**2 points**) We know that electrons move in circular orbits in the magnetic field. In the quantum mechanical picture, the impinging magnetic field B could be viewed as creating tiny whirlpools, so-called vortices, in the sea of electrons—one whirlpool for each flux quantum h/e of the magnetic field, where h is the Planck's constant and e the elementary charge of an electron. For the case of $R_{\mathrm{H}} = 3h/e^2$, which was discovered by Tsui and Stormer, derive the ratio of the number of the electrons N to the number of the flux quanta N_ϕ,

known as the filling factor ν.

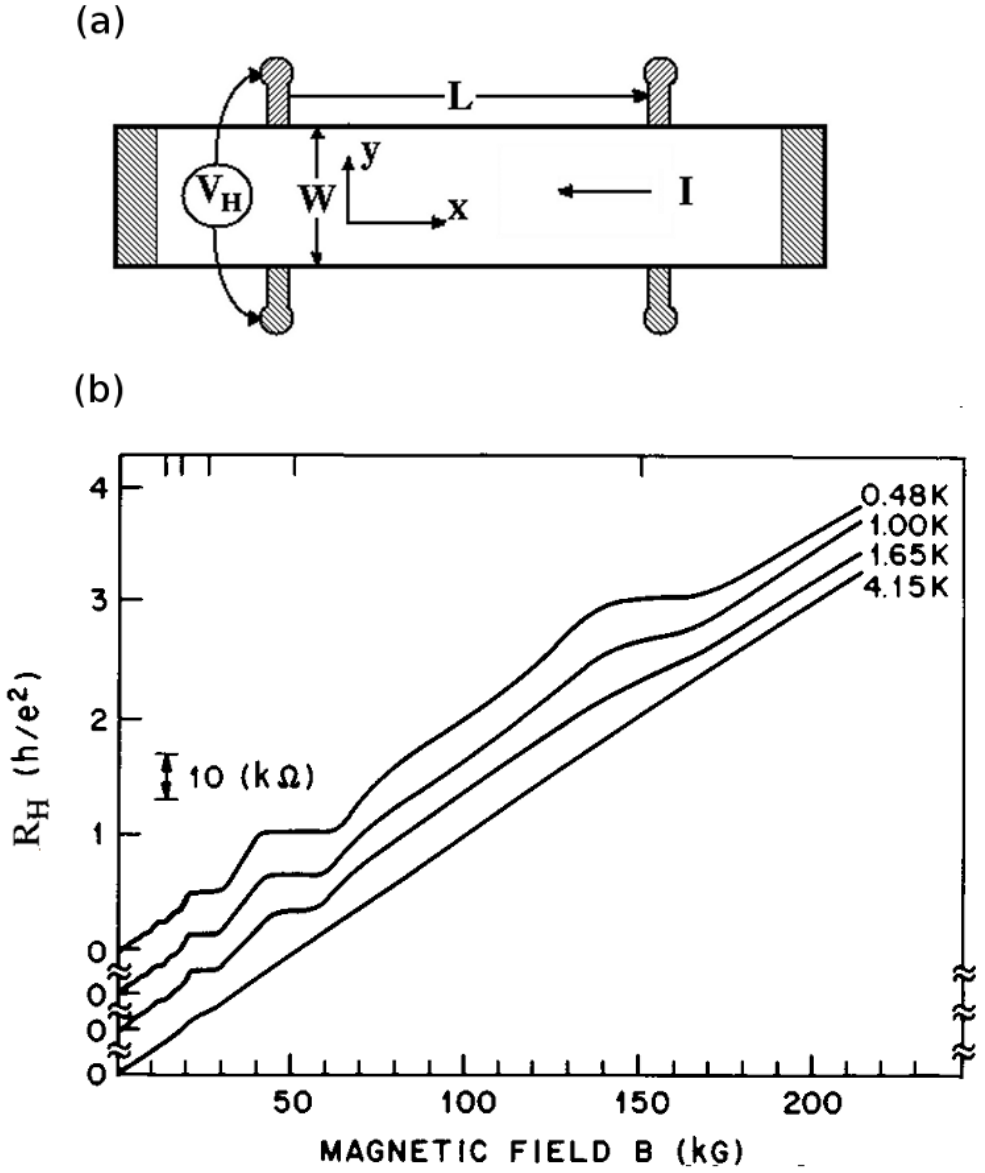


Figure 1: (a) Sketch of the experimental setup for the observation of the FQHE. As indicated, a current I is passing through a two-dimensional electron system in the longitudinal direction with an effective length L. The Hall voltage V_{H} is measured in the transverse direction with an effective width W. In addition, a uniform magnetic field B is applied perpendicular to the plane. The direction of the current is given for illustrative purpose only, which may not be correct. (b) Hall resistance R_{H} versus B at four different temperatures (curves shifted for clarity) in the original publication on the FQHE. The features at $R_{\mathrm{H}} = 3h/e^2$ are due to the FQHE.

(d) (**2 points**) It turns out that binding an integer number of vortices ($n > 1$) with each electron generates a bigger surrounding whirlpool, hence pushes away all other electrons. Therefore, the system can considerably reduce its electrostatic

Coulomb energy at the corresponding filling factor. Determine the scaling exponent α of the amount of energy gain for each electron $\Delta U(B) \propto B^{\alpha}$.

(e) (**2 points**) As the magnetic field deviates from the exact filling $\nu = 1/n$ to a higher field, more vortices (whirlpools in the electron sea) are being created. They are not bound to electrons and behave like particles carrying effectively positive charges, hence known as quasiholes, compared to the negatively charged electrons. The amount of charge deficit in any of these quasiholes amounts to exactly $1/n$ of an electronic charge. An analogous argument can be made for magnetic fields slightly below ν and the creation of quasielectrons of negative charge $e^* = -e/n$. At the quantized Hall plateau of $R_{\mathrm{H}} = 3h/e^2$, calculate the amount of change in B that corresponds to the introduction of exactly one fractionally charged quasihole. (When their density is low, the quasiparticles are confined by the random potential generated by impurities and imperfections, hence the Hall resistance remains quantized for a finite range of B.)

(f) In Tsui *et al.* experiment,

the magnetic field corresponding to the center of the quantized Hall plateau

$R_{\mathrm{H}} = 3h/e^2$, $B_{1/3} = 15$ Tesla,

the effective mass of an electron in GaAs, $m^* = 0.067\, m_e$,
the electron mass, $m_e = 9.1 \times 10^{-31}$ kg,
Coulomb's constant, $k = 9.0 \times 10^9$ N · m^2/C^2,
the vacuum permittivity, $\varepsilon_0 = 1/4\pi k = 8.854 \times 10^{-12}$ F/m,
the relative permittivity (the ratio of the permittivity of a substance to the vacuum permittivity) of GaAs, $\varepsilon_r = 13$,
the elementary charge, $e = 1.6 \times 10^{-19}$ C,
Planck's constant, $h = 6.626 \times 10^{-34}$ J · s, and
Boltzmann's constant, $k_{\mathrm{B}} = 1.38 \times 10^{-23}$ J/K.
In our analysis, we have neglected several factors, whose corresponding energy scales, compared to $\Delta U(B)$ discussed in (d), are either too large to excite or too small to be relevant.

(i) (**1 point**) Calculate the thermal energy E_{th} at temperature $T = 1.0$ K.

(ii) (**2 point**) The electrons spatially confined in the whirlpools (or vortices) have a large kinetic energy. Using the uncertainty relation, estimate the order of magnitude of the kinetic energy. (This amount would also be the additional energy penalty if we put two electrons in the same whirlpool, instead of in two separate whirlpools, due to Pauli exclusion principle.)

(g) There are also a series of plateaus at $R_{\mathrm{H}} = h/ie^2$, where $i = 1, 2, 3, \ldots$ in Tsui *et al.* experiment, as shown in Figure 1(b). These plateaus, known as the integer quantum Hall effect (IQHE), were reported previously by K. von Klitzing in 1980. Repeating (c)-(f) for the integer plateaus, one realizes that the novelty of the FQHE lies critically in the existence of fractionally charged quasiparticles. R.

de-Picciotto *et al.* and L. Saminadayar *et al.* independently reported the observation of fractional charges at the $\nu = 1/3$ filling in 1997. In the experiments, they measured the noise in the charge current across a narrow constriction, the so-called quantum point contact (QPC). In a simple statistical model, carriers with discrete charge e^* tunnel across the QPC and generate charge current I_B (on top of a trivial background). The number of the carriers n_τ arriving at the electrode during a sufficiently small time interval τ obeys Poisson probability distribution with parameter λ

$$\mathrm{P}(n_\tau = k) = \frac{\lambda^k \mathrm{e}^{-\lambda}}{k!}$$

where $k!$ is the factorial of k. You may need the following summation

$$\mathrm{e}^{\lambda} = \sum_{\mathrm{k}=0}^{\infty} \frac{\lambda^k}{k!}$$

(i) (**2 point**) Determine the charge current I_B, which measures total charge per unit of time, in terms of λ and τ.

(ii) (**2 points**) Current noise is defined as the charge fluctuations per unit of time. One can analyze the noise by measuring the mean square deviation of the number of current-carrying charges. Determine the current noise S_I due to the discreteness of the current-carrying charges in terms of λ and τ.

(iii) (**1 point**) Calculate the noise-to-current ratio S_I/I_B, which was verified by R. de-Picciotto *et al.* and L. Saminadayar *et al.* in 1997. (One year later, Tsui and Stormer shared the Nobel Prize in Physics with R. B. Laughlin, who proposed an elegant ansatz for the ground state wave function at $\nu = 1/3$.)

Question 2

How are aurora ignited by the solar wind?



Figure 1

The following questions are designed to guide you to find the answer one step by one step.

Background information of the interaction between the solar wind and the Earth's magnetic field

It is well known that the Earth has a substantial magnetic field. The field lines defining the structure of the Earth's magnetic field is similar to that of a simple bar magnet, as shown in Figure 2. The Earth's magnetic field is disturbed by the solar wind, whichis a high-speed stream of hot plasma. (The plasma is the quasi-neutralionized gas.)The plasma blows outward from the Sun and varies in intensity with the amount of surface activity on the Sun. The solar wind compresses the Earth's magnetic field. On the other hand, the Earth's magnetic field shields the Earth from much of the solar wind. When the solar wind encounters the Earth's magnetic field, it is deflected like water around the bow of a ship, as illustrated in Figure 3.

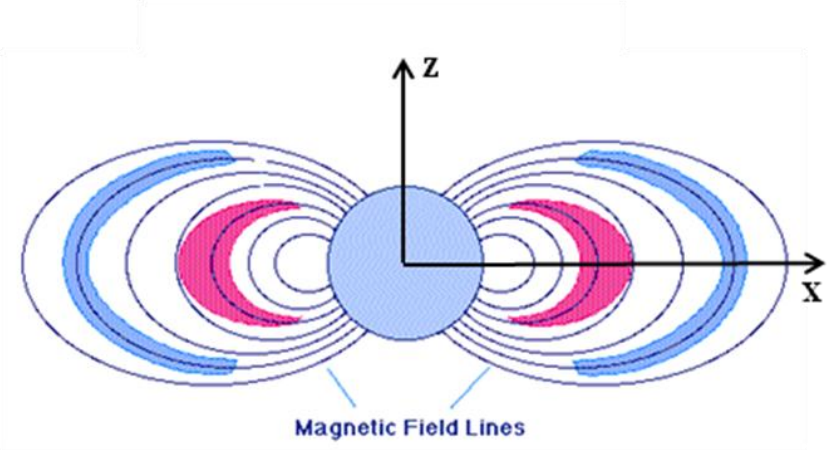


Figure 2

Question 2

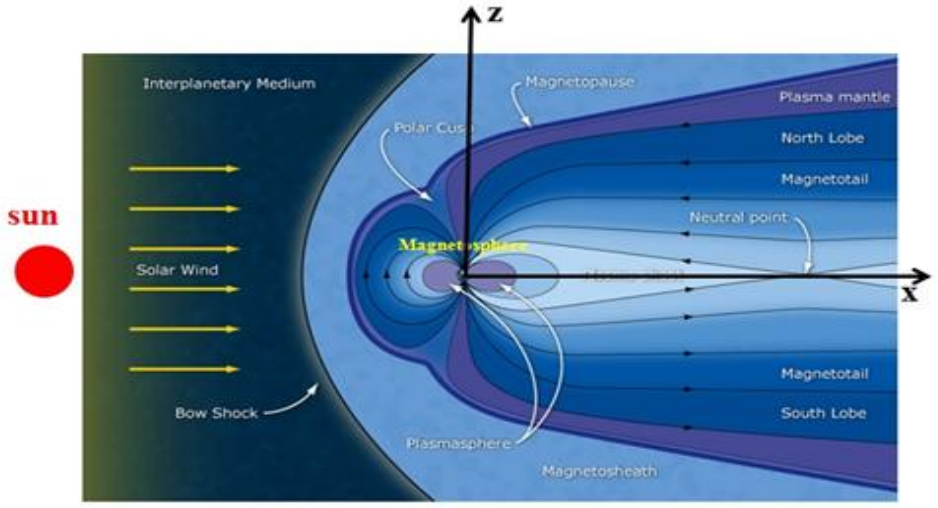
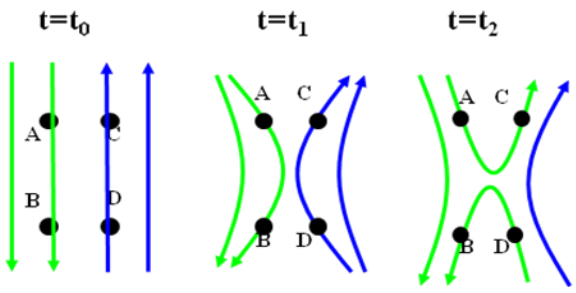


Figure 3

The curved surface at which the solar wind is first deflected is called the *bow shock*. The corresponding region behind the bow shock andfront of the Earth's magnetic field is called the*magnetosheath*. The region surroundedby the solar wind is called the *magnetosphere*.The Earth's magnetic field largely prevents the solar wind from enteringthe magnetosphere. The contact region between the solar wind and the Earth's magnetic field is named the *magnetopause*. The location of the magnetopause is mainly determined by the intensity and the magnetic field direction of the solar wind. When the magnetic field in the solar wind is antiparallel to the Earth's magnetic field, magnetic reconnection as shown in Figure 4 takes place at the dayside magnetopause, which allows some charged particles ofthe solar wind in the region "A" to move into the magnetotail "P" on the night side as illustrated in Figure 5. A powerful solar wind can push the dayside magnetopause tovery close to the Earth, which could cause a high-orbit satellite (such as a geosynchronous satellite) to be fully exposed to the solar wind. The energetic particles in the solar wind could damage high-tech electronic components in a satellite.Therefore, it isimportant to study the motion of charged particles in magnetic fields, which will give an answer of the aurora generation and could help us to understand the mechanism of the interaction between the solar wind and the Earth's magnetic field.



Question 2

Figure 4

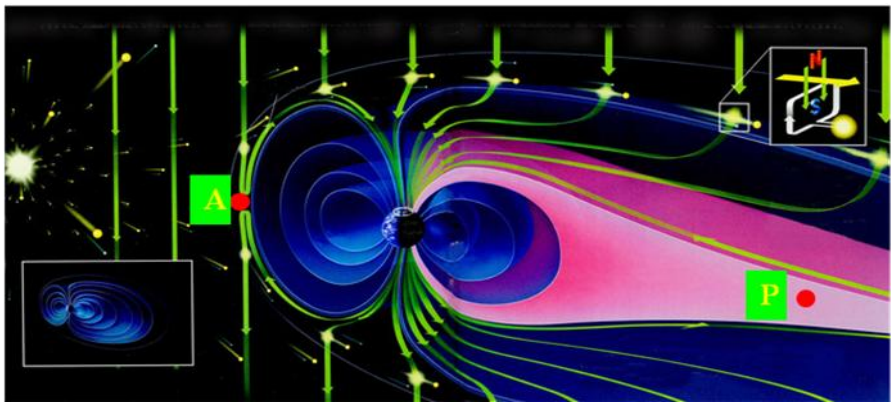


Figure 5

Numerical values of physical constants and the Earth's dipole magnetic field:

Speed of light in vacuum: $c = 2.998\times10^{8}\,\mathrm{m/s}$;

Permittivity in vacuum: $\varepsilon_0 = 8.9\times10^{-12}\,\mathrm{C^2/(N\cdot m^2)}$;

Permeability in vacuum: $\mu_0 = 4\pi\times10^{-7}\,\mathrm{N/A^2}$;

Charge of a proton: $e = 1.6\times10^{-19}\,\mathrm{C}$;

Mass of an electron: $m = 9.1\times10^{-31}\,\mathrm{kg}$;

Mass of a proton: $m_p = 1.67\times10^{-27}\,\mathrm{kg}$;

Boltzmann's constant: $k = 1.38\times10^{-23}\,\mathrm{J/K}$;

Gravitational acceleration: $g = 9.8\mathrm{m/s^2}$;

Planck's constant: $h = 6.626\times10^{-34}\,\mathrm{J\cdot s}$

Earth's radius $R_E = 6.4\times10^{6}\,\mathrm{m}$.

The Earth's dipole magnetic field can be expressed as

$$\vec{B}_d = \frac{B_0 R_E^3}{r^5}[3xz\hat{x} - 3yz\hat{y} + (x^2 + y^2 - 2z^2)\hat{z}] \quad ,(r \geq R_E) \qquad (1)$$

where $r = \sqrt{x^2 + y^2 + z^2}$, $B_0 = 3.1\times10^{-5}\,\mathrm{T}$, and $\hat{x}, \hat{y}, \hat{z}$ are the unit vectors in the x, y, z directions, respectively.

Questions:

(a) (3 Points)

(i) (1 Point) Before we study the motion of a charged particle in the Earth's dipole magnetic field,

Question 2

we first considerthe motion of an electron in a uniform magnetic field $\vec{B}$. When the initial electron velocity $\vec{v}$ is perpendicular to the uniform magnetic field as shown in Figure 6, please calculate the electron trajectory.The electron is initially located at (x,y,z)=(0,0,0).

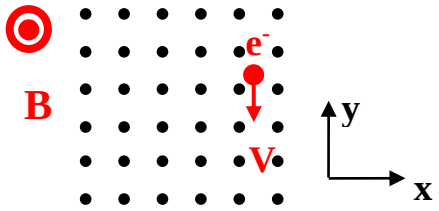


Figure 6

(ii) (1 Point) Please determine the electric current of the electron motion and calculate the magnetic moment $\vec{\mu} = I\vec{A}$, where $\vec{A}$ is the area of the electron circular orbit and the direction of $\vec{A}$ is determined by the right-hand rule of theelectric current.

(iii) (1 Point) If the initial electron velocity $\vec{v}$ is not perpendicular to the uniform magnetic field, i.e, the angle θ between $\vec{B}$ and $\vec{v}$ is $0^0 < \theta < 90^0$, please give the screw pitch (the distance along the z-axis between successive orbits) of the electron trajectory.

(b) (4 Points) In the uniform background magnetic field as shown in Figure 6, theplasma density isnonuniform in *x*. For simplicity, we assume that the temperature and the distribution of the ions andelectrons are the same. Thus, the plasma pressure can be expressed as

$$p(x) = kT[n_i(x) + n_e(x)] = 2kTn(x) = 2kT(n_0 + \alpha x),$$

Where *B*, *T*, *k*, n_0, and α are positive constants, $n_i(x)$ and $n_e(x)$ are the number densities of the ions and electrons.

(i) (2 Points) Please explain the generation mechanism of the electric current by a schematic drawing.

(ii) (2 Points) If both the ions and electrons have a Maxwellian distribution, the ion distribution is

$$f_i(x, v_\perp, v_\parallel) = n_i(x)\left(\frac{m_i}{2\pi kT}\right)^{3/2} e^{-m_i(v_\perp^2 + v_\parallel^2)/2kT},$$

please calculate the constant β in the magnetization $M = \beta n(x)\frac{kT}{B}$, where the magnetization

Question 2

*M*is the magnetic moment per unit volume. (Hint: We have $\int_0^\infty x\exp(-x)dx = 1$ and

$\int_{-\infty}^{\infty} \exp(-x^2)dx = \sqrt{\pi}$.)

(c) (1 Point)Now let's go back to the Earth's dipole magnetic field. Please apply the result from Question(b) to calculate the ratio of the diamagnetic field and the Earth's dipole magnetic field in Equation (1) at the position (x=10R_E，y=0, z=1 R_E). The plasma pressure is assumed to be $p(z) = p_0 e^{-(z/a)^2}$, where $p_0 = 3\times10^{-10}$pa and $a = 2R_E$ .The magnetic field around this position is also assumed to be uniform. Be aware of the difference in the coordinate systemsin Questions (b) and (c). (Hint: The diamagnetic field is given by $B_{mx} = \mu_o M$.)

(d) (4 Points) From Figures 2, 3, and 5, it can be clearly seen that the Earth's magnetic fieldstrength along a magnetic field line is the largest at the poles and the smallest in the equatorial plane.Since the Earth's dipole magnetic field is axially symmetric and slowly varying along a magnetic field line, it can for simplicitybe treated as a magnetic-mirror field as shown in Figure 7.The magnetic field strength along a magnetic field line is the smallest (B_0) at the point "P_2" and the largest (B_m) at the points "P_1" and "P_3". An electron with an initial velocity $\vec{v}$ is located at the point "P_2" and drifts towards the point "P_3". The angle between the initial velocity $\vec{v}$ and the magnetic field at the point "P_2" is $0^0 < \theta < 90^0$. For the magnetic-mirror field $\vec{B} = B_r\hat{r} + B_z\hat{z}$ (with $B_r << B_z$), we can assume $\frac{dB}{dz} = \frac{dB}{ds}$, where $\frac{dB}{ds}$ is the spatial derivative of B along a magnetic field line. Since there is no evidence of the existence of magnetic monopoles, we have $\langle B_r \rangle = -\frac{1}{2}\frac{dB}{dz} r_c << B_z$, where $\langle B_r \rangle$ is the gyro-average of B_r and r_c is the electron gyroradius.

Question 2

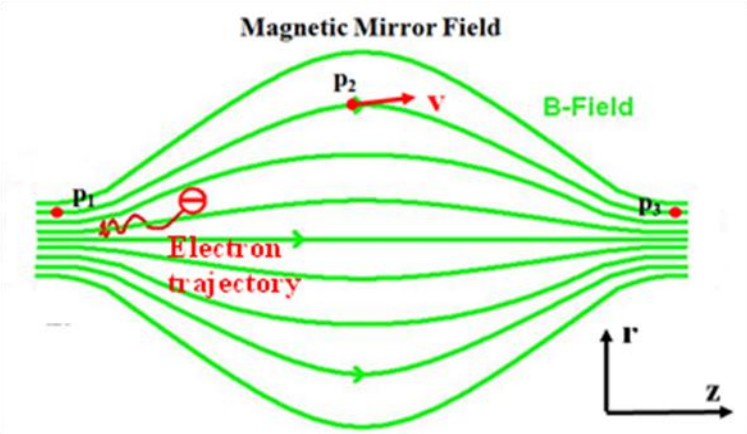
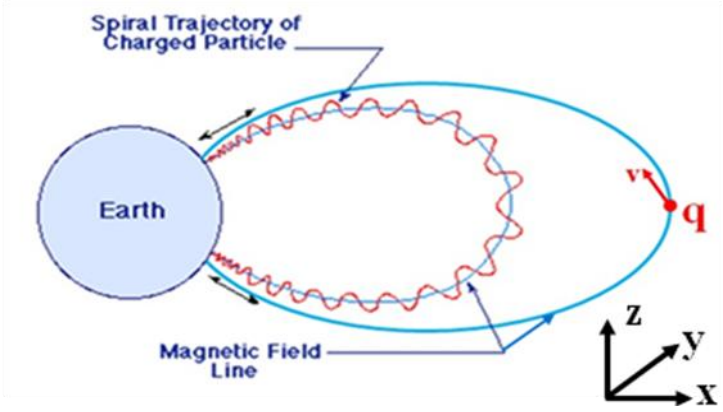


Figure 7

(i) (3 Points) Please givethe gyro-averaged magnetic-field force along the magnetic field lines on an electron and show that the magnetic moment is a motion constant, i.e., $\frac{d\mu}{dt}=0$, based on the law of the total kinetic energy conservation.

(ii) (1 Point) Based on the motion constant of magnetic moment, please determinewhat the condition should be satisfied for the angle θ between the initial electron velocity $\vec{v}$ and the magnetic field at the point "P_2"ifan electron will not escape from the magnetic mirror field.

(e) (1 Point) Earth's dipole magnetic field lines (blue lines) are shown in Figure 8. The spiral trajectory of a charged particle (red curve) is assumed to be confined in the y=0 plane since the gradient and the curvature of the magnetic field can be ignored. If a charged particle with the mass m, charge q, and velocity $\vec{v}$ is initially located at the equatorial position [x=6R_E, y=0, z=0] and the angle between the electron velocity $\vec{v}$ and the magnetic field is θ initially, please determine what the condition should be satisfied for θ if the charged particle arrives below 200km of its altitude at the latitude 60^0.



Question 2

Figure 8

(f)(5 Points) As shown in Figure 5, when magnetic reconnection takes place at the dayside magnetopause, reconnected magnetic field lines drift towards the nightside region because the solar wind flows tailward. Thus, some solar wind electrons in the region "A" also move towards the magnetotail in the region "P". After the electrons arrive in the region "P", some electrons can be accelerated to around 1keV. If energetic electrons drift down to the thermosphere (The altitude of the thermosphere is about 85km-800km.), energetic electrons can collide with the neutral atoms, which could cause the neutral atoms to jump into excited states. A photon is emitted when the higher excited state of aneutral atom returnsto its lower excited state or ground state. Splendid aurora (Figure 1) is generated in the aurora oval due to photons with different wavelengths. It is found that the aurorais mainly resulted fromphotonsemittedby oxygen atoms.The energy levels in the first and second excited states relative to the ground stateare 1.96eV and 4.17eV, respectively. The lifetimes of the two excited states of an oxygen atomare 110s and 0.8s as shown in Figure 9.

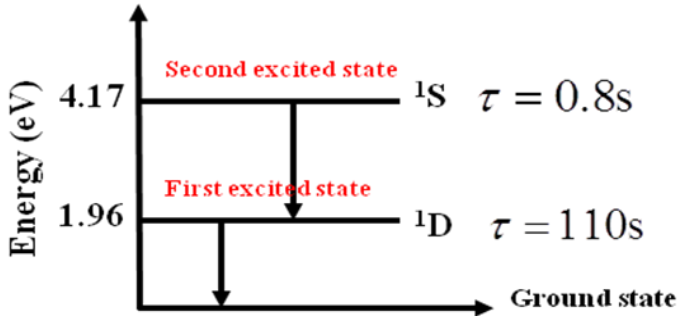


Figure 9

(i) (2 Points) Please give the atmospheric density as a function of the altitude and the ratio of the oxygen density at the altitudes H=160km and H=220km. For simplicity, we assume that the atmospheric temperature is independent of the altitude and the air is an ideal gas.

($\rho_0 g / P_0 = 0.13 / \text{km}$, where ρ_0 and P_0 are the atmospheric density and pressure at sea level.)

(ii) (3 Points)Please give the colors of auroras at the altitudes H=160km and H=220km. (Hint: The dependence of the collision frequency of atmosphericmolecules on the atmospheric density is $\nu = \nu_0 \rho / \rho_0$, where $\nu_0 \approx 10^9 / s$ is the collision frequency of atmosphericmolecules at sea level.The excited oxygen atom will lose a part of its energy when it collides with other neutral molecules.)

(g) (2 Points) As mentioned above, a powerful solar wind can push the dayside magnetopause tovery close to the Earth, which could cause a high-orbit satelliteto be fully exposed to the solar wind. The energetic particles in the solar wind could damage high-tech electronic components in a

Question 2

satellite.For simplicity, the Earth's dipole magnetic field is assumed to remain unchanged when the solar wind compresses it and that the plasma density is ignorable in the magnetosphere. Please give the minimum solar wind speed to cause a damage of a geosynchronous satellite if the magnetic field strength and the plasma density of the solar wind are $B_s = 5\times10^{-9}\,\mathrm{T}$ and $\rho_s = 50\ \mathrm{proton/cm^3}$, respectively. (Hint: The force per unit area associated with the magnetic field is $f = B^2/2\mu_0$. We only consider the variation in *x* for all physical quantities, i.e., the physical quantities are independent of *y* and *z*.)

Question 3

Figure 1 shows a Fabry-Perot (F-P) etalon, in which air pressure is tunable. The F-P etalon consists of two glass plates with high-reflectivity inner surfaces. The two plates form a cavity in which light can be reflected back and forth. The outer surfaces of the plates are generally not parallel to the inner ones and do not affect the back-and-forth reflection. The air density in the etalon can be controlled. Light from a Sodium lamp is collimated by the lens L1 and then passes through the F-P etalon. The transmitivity of the etalon is given by $T=\frac{1}{1+F\sin^2(\delta/2)}$, where $F=\frac{4R}{(1-R)^2}$, R is the reflectivity of the inner surfaces, $\delta=\frac{4\pi nt\cos\theta}{\lambda}$ is the phase shift of two neighboring rays, n is the refractive index of the gas, t is the spacing of inner surfaces, θ is the incident angle, and λ is the light wavelength.

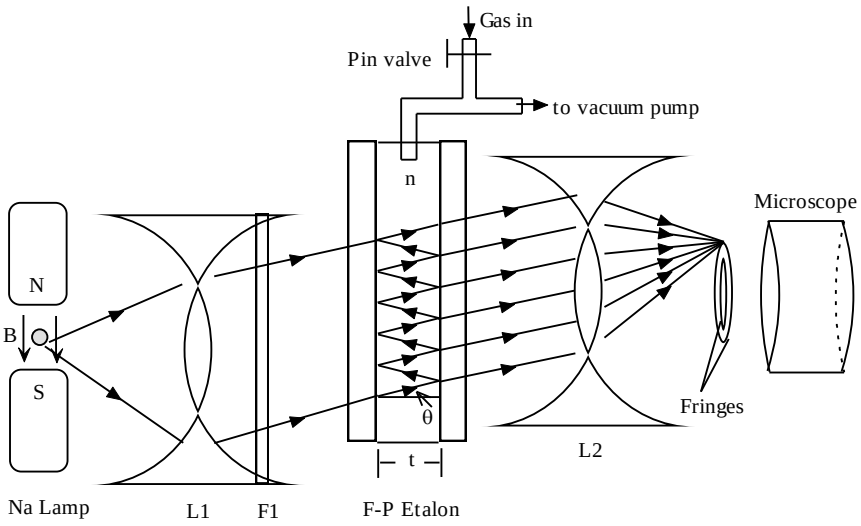


Figure 1

The Sodium lamp emits D1 ($\lambda=589.6nm$) and D2 ($589nm$) spectral lines and is located in a uniform magnetic field. For simplicity, an optical filter F1 is assumed to only allow the D1 line to pass through. The D1 line is then collimated to the F-P etalon by the lens L1. Circular interference fringes will be present on the focal plane of the lens L2 with a focal length f=30cm. Different fringes have the different indident angle θ. A microscope is used to observe the fringes. We take the reflectivity R= 90% and the inner-surface spacing t=1cm.

Some useful constants : $h=6.626\times10^{-34}J\cdot s$, $e=1.6\times10^{-19}C$, $m_e=9.1\times10^{-31}kg$, $c=3.0\times10^{8}ms^{-1}$.

（a）（**3points**）The D1 line ($\lambda = 589.6nm$) is collimated to the F-P etalon. For the vacuum case (n=1.0), please calculate (i) interference orders m_i, (ii) incidence angle θ_i and (iii) diameter D_i for the first three（i=1，2，3）fringes from the center of the focal plane.

（b）（**3 points**）As shown in Fig. 2, the width ε of the spectral line is defined as the full width of half maximum of the phase shift δ. The resolution of the F-P etalon is defined as follows: for two wavelengths λ and $\lambda + \Delta\lambda$, when the central phase difference $\Delta\delta$ of both spectral lines is larger than ε, they are thought to be resolvable; then the etalon resolution is $\lambda / \Delta\lambda$ when $\Delta\delta = \varepsilon$. For the vacuum case, the D1 line ($\lambda = 589.6nm$), and the incident angle $\theta = 0$, please calculate：

（i）the width ε of the spectral line.

（ii）the resolution $\lambda/\Delta\lambda$ of the etalon.

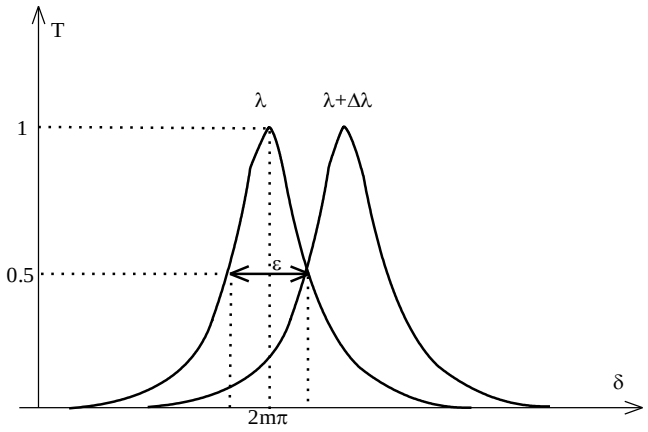


Figure 2

（c）（**1 point**）As shown in Fig. 1, the initial air pressure is zero. By slowly tuning the pin valve, air is gradually injected into the F-P etalon and finally the air pressure reaches the standard atmospheric pressure. On the same time, ten new fringes are observed to produce from the center of the focal plane. Based on this phenomenon, calculate the refractive index of air n_{air} at the standard atmospheric pressure.

（d） **(2 points)** Energy levels of Sodium atoms occurs when they are placed in a magnetic field. This is called as the Zeeman effect. The energy shift given by $\Delta E = m_j g_k \mu_B B$， where the quantum number m_j can be J，J-1，…，-J+1，-J，J is the total angular quantum number, g_k is the Landé factor, $\mu_B = \dfrac{he}{4\pi m_e}$ is Bohr magneton，h is the Plank constant，e is the electron charge，m_e is the electron mass, B is the magnetic field. As shown in Fig. 3, the D1 spectral line is emitted

when Sodium atoms jump from the energy level $^2P_{1/2}$ down to $^2S_{1/2}$. We have $J=\frac{1}{2}$ for both $^2P_{1/2}$ and $^2S_{1/2}$. Therefore, in the magnetic field, each energy level will be split into two levels. We define the energy gap of two splitting levels as ΔE_1 for $^2P_{1/2}$ and ΔE_2 for $^2S_{1/2}$ respectively ($\Delta E_1 < \Delta E_2$). As a result, the D1 line is split into 4 spectral lines (a, b, c, and d), as showed in Fig. 3. Please write down the expression of the frequency (ν) of four lines a, b, c, and d.

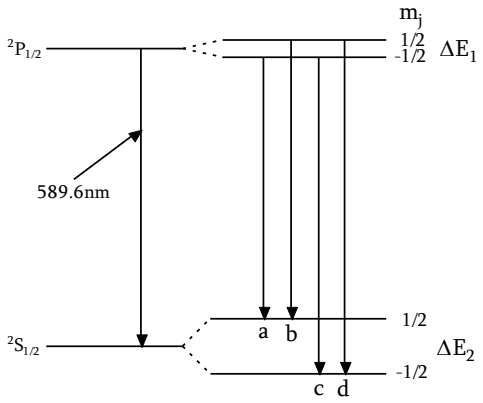


Figure 3

(e) **(3 points)** As shown in Fig. 4, when the magnetic field is turned on, each fringe of the D1 line will split into four sub-fringes (1, 2, 3, and 4). The diameter of the four sub-fringes near the center is measured as D_1, D_2, D_3, and D_4. Please give the expression of the splitting energy gap ΔE_1 of $^2P_{1/2}$ and ΔE_2 of $^2S_{1/2}$.

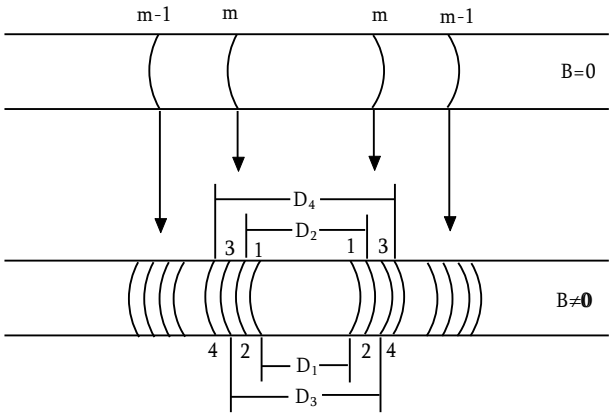


Figure 4

f） （**3 points**）For the magnetic field B=0.1T，the diameter of four sub-fringes is measured as：$D_1 = 3.88mm$， $D_2 = 4.05mm$， $D_3 = 4.35mm$，and $D_4 = 4.51mm$. Please calculate the Landé factor g_{k1} of $^2P_{1/2}$ and g_{k2} of $^2S_{1/2}$.

g） **(2 points)** The magnetic field on the sun can be determined by measuring the Zeeman effect of the Sodium D1 line on some special regions of the sun. One observes that, in the four split lines,

the wavelength difference between the shortest and longest wavelength is 0.012nm by a solar spectrograph. What is the magnetic field B in this region of the sun?

(h)（**3 points**） A light- emitting diode (LED) source with a central wavelength $\lambda = 650nm$ and spectral width $\Delta\lambda = 20nm$ is normally incident ($\theta = 0$) into the F-P etalon shown in Fig. 1. For the vacuum case, find (i) the number of lines in transmitted spectrum and (ii) the frequency width $\Delta\nu$ of each line?

16th ASIAN PHYSICS OLYMPIAD 2015
3rd-11th MAY, HANGZHOU, CHINA

Experimental Competition

May 7, 2015

08:30-13:30 hours

Experiment Problem

There are 22 pages including the cover page.

Please first read the following instructions carefully:

1. The time available is 5 hours for the experimental problem.
2. Use only the pen and equipment provided.
3. There is a set of Answer Sheets in which you have to enter your data and results.
4. Write down your Student Code in the boxes at the top of each Answer Sheets and additional sheets which you submit.
5. Additional Writing Sheets are provided.
6. If you use any additional Writing Sheets, please write down your **Student Code**, the **Question Number** and the **Page Number** on these additional Writing Sheets.
7. If you use additional Writing Sheets that you do not wish to be marked, put a large '**X**' across the entire sheet.
8. You should use mainly equations, numbers, symbols, graphs, figures and as little text as possible in your answers. Use the symbols defined in the question.
9. At the end of the experiment arrange all sheets in the following order:
   a. Main Answer Sheets
   b. Used Writing Sheets
   c. Writing sheets which are marked with '**X**'
   d. Unused Writing Sheets
   e. The question paper
10. Put all the papers inside the envelope and leave the envelope on your desk.
11. **You are not allowed to take any sheet of paper or any material used in the experiment out of the room.**

The Piezoelectric Effect And Its Applications

1. Introduction

The piezoelectric effect refers to the process that the electric charge accumulates in solid materials in response to applied mechanical stress (see Figure 1(a)). It is reversible, which means that materials exhibiting the piezoelectric effect also exhibit the converse piezoelectric effect, i.e., the internal generation of a mechanical strain resulting from an applied electrical field (see Figure 1(b)).

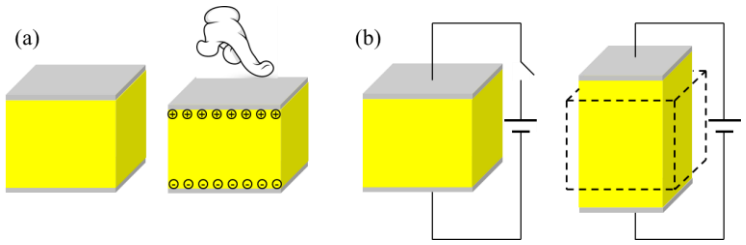


Figure 1. (a) The piezoelectric effect. Left: a yellow piezoelectric cube under no mechanical stress. Right: the electric charge accumulates on opposite surfaces of the cube in response to applied mechanical stress. (b) The converse piezoelectric effect. Left: without applying an electric field, the cube is un-stressed and remains in its natural shape. Right: the cube is stressed and deformed resulting from an applied electric field.

The piezoelectric materials are used in various applications covering a wide range from industrial and manufacturing to daily life, such as the production and detection of sound, generation of high voltages, microbalances, ultra-fine focusing of optical assemblies, ignition source for cigarette lighters, push-start propane barbecues, and quartz watches.

In addition to applications mentioned above, the piezoelectric materials are also actively used in scientific research. As very high electric fields correspond to only tiny changes in the dimensions of the piezoelectric materials, the piezoelectric materials become the most important tool for positioning objects with extreme accuracy. They are the basis of some commonly used tools in surface science, the scanning tunneling microscope (STM) and its variants. The 1986 Nobel Prize in physics was awarded to Gerd Binnig and Heinrich Rohrer for their design of STM.

Another merit of the piezoelectric materials is that they could enable conversion of signals between different modes such as mechanical, electrical and optical. With the help of ultra-low temperatures and state-of-the-art electronics, researchers are able to cool down the mechanical mode to its ground state and observe the quantization of motion. The experiment of creating such a quantum machine, a mechanical resonator made by piezoelectric aluminum nitride, was titled “Breakthrough of the Year 2010” by Science magazine.

There are many piezoelectric materials, both natural and synthetic. Some naturally occurring materials include quartz, bone and silk. Synthetic materials include ceramics, semiconductors and polymers. Lead zirconate titanate ($Pb[Zr_xTi_{1-x}]O_3$), known as PZT, is the most common piezoelectric ceramic in use

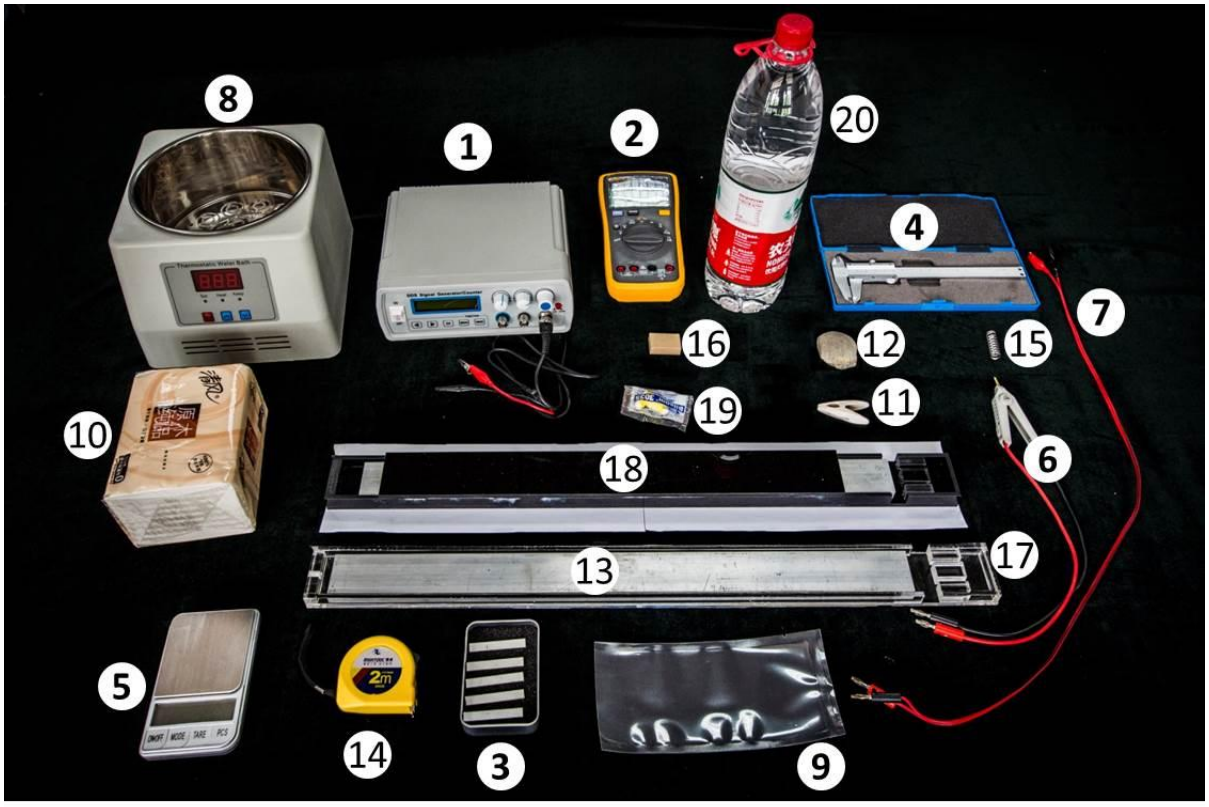
today, which exhibits strong piezoelectricity.

In this APhO2015 experiment, we will explore properties of PZT and its applications. For a given PZT plate, we will measure its piezoelectric coefficient via the resonant method and will estimate its Curie temperature by linear extrapolation. We will make a transducer out of a PZT plate to produce mechanical motions and sound waves in the medium; we will make a sensor out of a PZT plate for sensing the strength of sound waves. With the hand-made transducer and senor, we will measure the longitudinal and transverse wave velocities of sound in an aluminum rod. Finally, we will use sound waves to resonantly locate an artificially-designed defect in another aluminum rod.

2. General Safety Precautions:

1) **Be sure to switch off the equipment before plug in/out its power cord. Otherwise damage can occur.**
2) **Do not turn on the thermostat water bath if the heating unit is not covered by water.**
3) **Be careful not to spill the water onto nearby electronics and the electrical power socket.**
4) **Be careful of the hot water.**
5) **Be careful of electric shock.**
6) **Do not drink/consume any of the materials provided for the experiment.**

3. Apparatus



1) A signal generator which can output simple repetitive electrical waveforms over a wide range of frequencies.
2) A digital multimeter (DMM).
3) 5 PZT plates. Two flat surfaces of each plate are coated by thin films of silver.
4) A Vernier caliper.
5) An electronic weighing scale.
6) A Kelvin clip. The Kelvin clip features a crocodile clip with two isolated jaws, which are connected to two banana plugs respectively. It is used to clamp a PZT plate.
7) A cable with two banana plugs connecting to two crocodile clips, respectively. One jaw of each crocodile clip is wrapped by a black tube, and therefore it is important to have the correct polarity when clamping.
8) A thermostat water bath.
9) A plastic bag.
10) Paper towels.
11) A plastic clip.
12) A pebble.
13) An aluminum rod.
14) A steel tape measure.
15) A spring.
16) An eraser.

17) A transparent plastic box to accommodate the aluminum rod and the PZT plates together.
18) A black plastic box with an aluminum rod inside. A defect, invisible from outside the box, is artificially engineered at a spot along the rod.
19) A pair of earplugs.
20) 1.5 L bottled water.

Instructions for the electronic weighing scale (see Figure 2)

- Place the scale on a flat, very stable surface.
- Press the “ON/OFF” button to turn on the scale.
- Wait until the reading is stable. If the reading is not zero, press the “TARE” button to re-Zero it.
- Press the “MODE” button to toggle units between “g”, “gn”, “oz”, “ozt”, “dwt”, “ct” and “tl”. It is recommended that you use the unit “g” (gram).



Figure 2. An electronic weighing scale.

Instructions for the signal generator (see Figure 3)

- To turn on the machine, connect the detachable USB power cord (with the AC adapter) to the rear panel receptacle and turn on the front panel power button.
- The "Display Panel" shows the wave frequency and the wave type (sine, square, or triangle). We recommend sine for the experiment.
- Use the "Amplitude" knob to adjust the signal amplitude. Use the "Adjust" knob to change the signal frequency. Use the "◀" or "▶" button to move the cursor.
- **Be careful when tuning the "DC offset" knob. This knob changes the DC offset of the signal. A big DC offset may cause signal clipping (see Figure 4 (a)). It is recommended that you calibrate the DC offset before using the signal generator: while using the DMM to monitor the DC voltage of the output, adjust the "DC Offset" knob until the DC voltage reaches zero.**
- **It is also recommended that you do not tune the "Amplitude" knob to maximum to avoid signal clipping (see Figure 4 (b)). You can tune the output amplitude to 3.0 V (rms) for the experiment: while using the DMM to monitor the AC voltage of the output at a frequency of, e.g., 1 kHz, adjust the "Amplitude" knob until the AC voltage reaches about 3.0 V (rms).**
- If you press a button by mistake and do not know how to return to the original configuration, restart the machine to restore to default configuration.

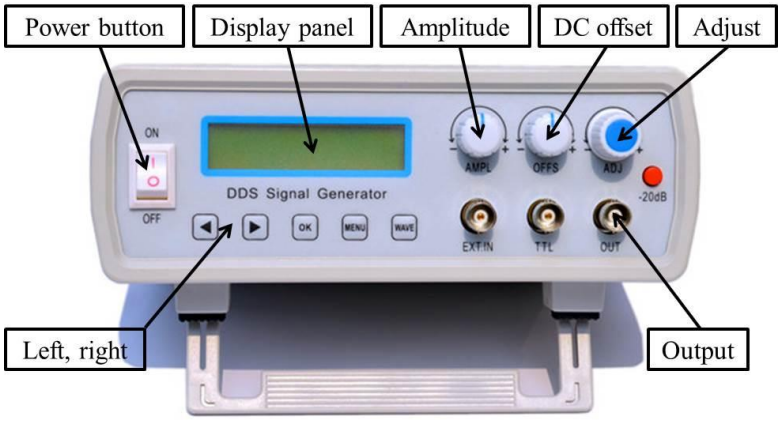


Figure 3. A signal generator.

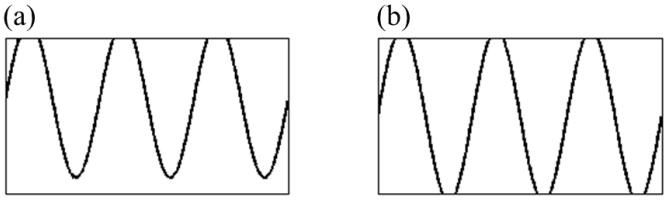


Figure 4. Two symptoms of signal clipping. (a) Signal clipping when the DC offset is nonzero. (b) Signal clipping when the output amplitude is too large.

Instructions for the digital multimeter (DMM. See Figure 5)

- Use the "VΩ" and "COM" inlets for measuring voltage, resistance and capacitance.
- Use the "mA" and "COM" inlets for measuring current.
- Use the rotary switch to select the proper function and measuring range.
- Toggle between the AC and DC modes by pressing the YELLOW button.
- The DMM enters the "Sleep mode" and blanks the display if the DMM remains inactive for more than 20 minutes. Turn the rotary switch to OFF and back to wake up the DMM. To disable the Sleep mode, hold down the YELLOW button while turning the DMM on.
- **Attention: although it is usable for the experiment with frequencies up to 40 kHz, the DMM is not designed for accurately measuring the amplitude values of AC signals above 1 kHz. To calibrate the output voltage of the signal generator using the DMM, you should set the signal frequency to 1 kHz or below.**

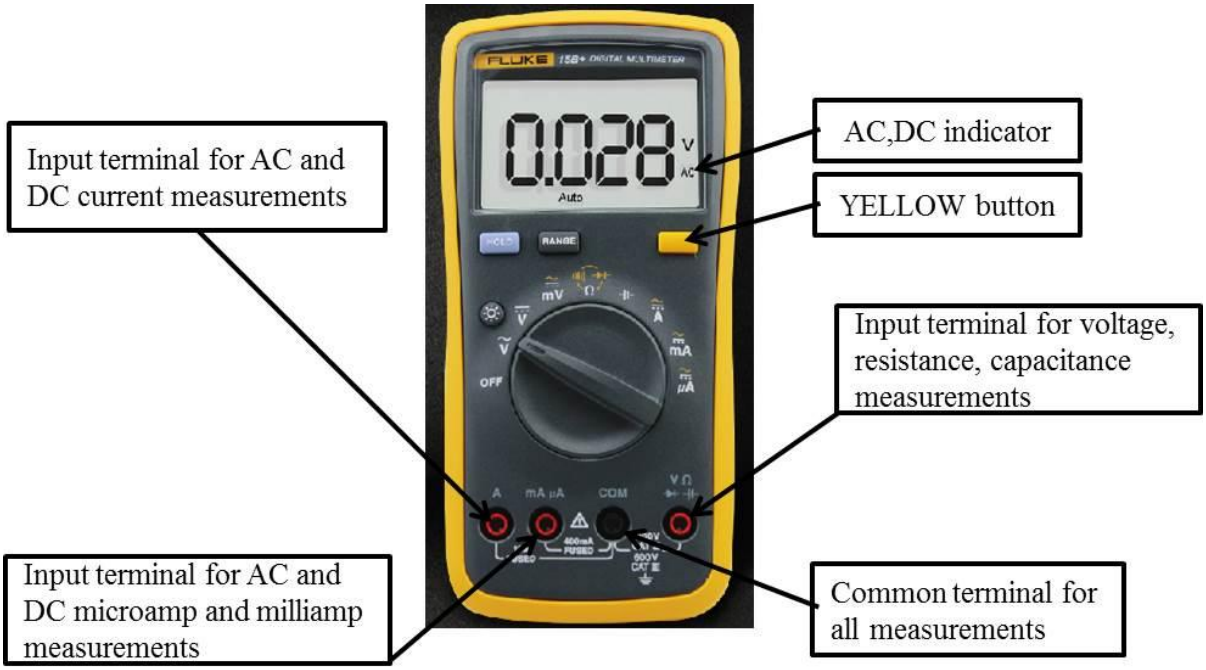


Figure 5. A digital multimeter.

Instructions for the thermostat water bath (see Figure 6)

- **Surfaces can become hot during use.**
- **It is strictly prohibited to turn on the machine if the heating unit is not covered by water.**
- **Be careful not to spill water onto the nearby electronics and the power socket.**
- Fill in bottled water for the bath to be about half full. Properly connect the power cord and turn the bath on.
- To set the target temperature, press the "Set" button to enter the "Set" mode and the "Set" indicator will illuminate. Use the "Increasing" ("Decreasing") button to increase (decrease) the displayed value to the target temperature. Press the "Set" button again to exit the "Set" mode and the water bath will start heating automatically. Check the "Temperature display" for actual temperature readings.
- During heating, the "Heat" indicator illuminates. After reaching the set temperature, the "Keep" indicator will illuminate and heating will stop.
- It is recommended that you ramp up the temperature gradually from low to high during the experiment.

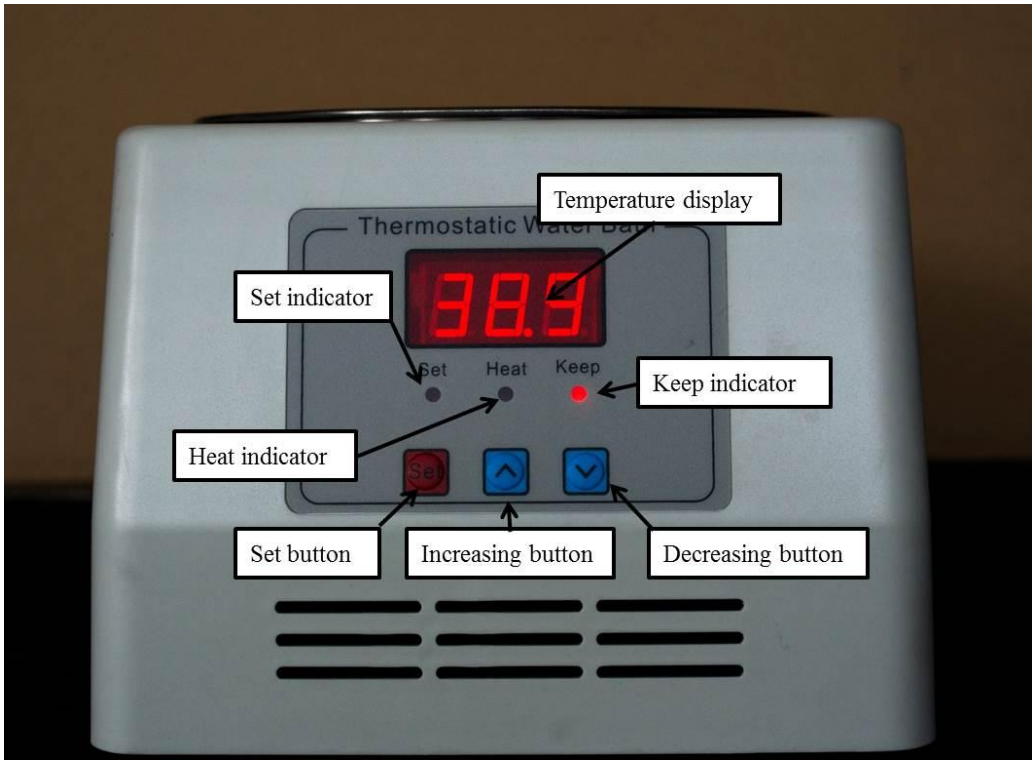


Figure 6. A thermostat water bath.

Experiment A

Basic measurement [3.0pts]

In this experiment, you are required to measure the dimensions, the mass and the capacitance of a PZT plate, and then calculate its density ρ and relative permittivity ε_r.

Please choose a PZT plate. You are supposed to perform Experiments A, B and C using this same plate.

In Experiments A to E, error analysis is required if it is explicitly stated; it is not required if it is not stated.

A.1	Choose a PZT plate and use the Vernier caliper to measure its length l, width w, and thickness t. Use the electronic weighing scale to measure its mass m. Use the DMM and the Kelvin clip to measure its capacitance C (at ambient temperature). Considering the slight non-uniformity in the dimensions of the PZT plate and the uncertainties of instrumental readings, repeat each measurement several times and then calculate the mean and the standard error.	1.6pts

Attention: The relative permittivity of the PZT plate is temperature dependent (see **Experiment C**). You are supposed to perform the capacitance measurement at ambient temperature. Avoid the direct warming up of the plate by your hand.

A.2	Now calculate the density ρ and the relative permittivity ε_r of the PZT plate. Based on standard errors obtained from A.1, carry out the error analysis to estimate the uncertainties of ρ and ε_r (vacuum permittivity $\varepsilon_0 = 8.85\times10^{-12}$ F/m).	1.4pts

Experiment B

The resonant method to measure the piezoelectric coefficient [4.5pts]

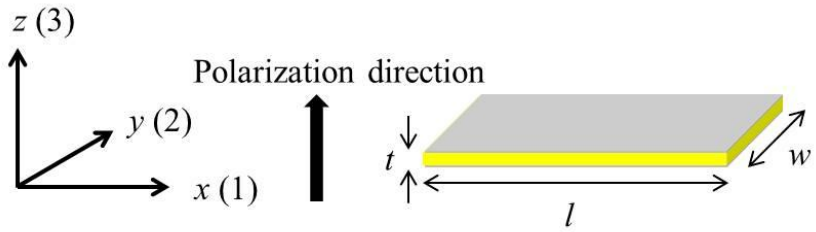


Figure 7. The PZT plate.

As described in the introduction section, the piezoelectric plate produces distortion (also called strain *S*) when subjected to an electric field. The proportional coefficient *d* of the strain *S* versus the electric field strength *E* is defined as the piezoelectric coefficient

$$d = \frac{S}{E}.$$

In reality, the PZT plate is anisotropic. There is a special direction called the polarization direction. During production of the PZT plate, a strong DC electric field is applied along its thickness direction (z-axis in Figure 7) to align the molecular dipoles of the ceramic at a temperature higher than the Curie temperature (see **Experiment C**). This polarization remains after temperature is reduced below the Curie temperature and then the DC electric field is removed.

The top and bottom flat surfaces of the plate are coated with silver films as electrodes (see Figure 7). The electric field is along z-axis (3) when the electrodes are connected to a voltage source, and we shall denote it as E_3. Here we define

$$d_{31} = \frac{S_1}{E_3},$$

$$d_{33} = \frac{S_3}{E_3},$$

where $S_1 = \Delta l / l$ and $S_3 = \Delta t / t$ are strains along x-axis (1) and z-axis (3), respectively.

Note that the strain is not necessarily parallel to the electric field E_3. For PZT materials, d_{31} is roughly half of d_{33}. According to the parameters in **Experiment A**, it can be shown that length *l* changes the most when a voltage *V* is applied across the electrodes, i.e.,

$$\Delta l = l d_{31} E_3 = \frac{l}{t} d_{31} V,$$

$$\Delta w = \frac{w}{t} d_{31} V,$$

$$\Delta t = t d_{33} E_3 = d_{33} V = 2 d_{31} V,$$

where $l/t >> w/t >> 2$. **To simplify the theoretical treatment, for such a long thin plate, vibrations along the width (y-axis) and thickness (z-axis) directions can be neglected and the problem reduces to a one-dimensional vibration problem. As such we remove the redundant subscript and simply denote d_{31} as d.** In **Experiments D** and **E** the ignored vibration along the width (y-axis) direction may cause slight imperfection to the measurement**.**

The PZT plate performs like a pure capacitor (with a capacitance C from A.1) when driven by low-frequency signals. However, as frequency increases, the vibration of the PZT plate changes its circuitry behavior significantly. At certain frequencies called **resonant frequencies**, the plate vibrates strongly and its impedance reaches a minimum. Along with the resonant frequencies, there are also frequencies where the impedance reaches a maximum, and we call them **antiresonant frequencies**.

The first resonant frequency f_r of the plate is associated with its fundamental vibration mode along the length direction (x-axis). Near f_r, the PZT plate can be approximated by a simple circuit, with two capacitors (C_0 and C_1) and an inductor (L_1) being arranged as shown in Figure 8.

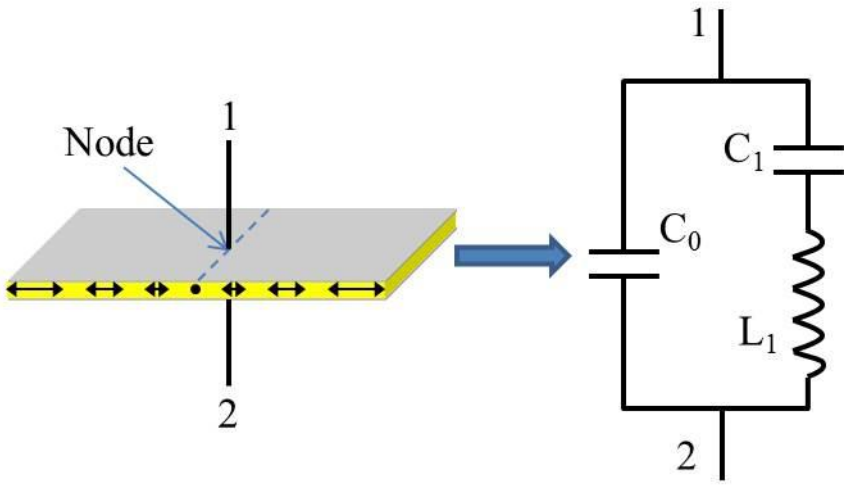


Figure 8. The equivalent circuit model (in response to an external signal drive) of the PZT plate near its first resonant frequency. The PZT plate vibrates in its fundamental mode. Under the free boundary condition, the middle point along the length direction is the node.

B.1	Derive the expressions for the resonant frequency f_r and the antiresonant frequency f_a of the equivalent circuit.	1.0pts

The piezoelectric coefficient d can be calculated by the following formula

$$d = \sqrt{\frac{\varepsilon_0 \varepsilon_r}{128 f_r^4 l^2 \rho [\frac{1}{(2\pi f_a)^2 - (2\pi f_r)^2} + \frac{1}{32 f_r^2}]}}.$$

Now we perform the experiment to locate f_r and f_a. See the circuit schematics in Figure 9. We keep the output amplitude (the voltage V) of the signal generator constant, such that the impedance of the PZT plate correlates with the AC current in the circuit.

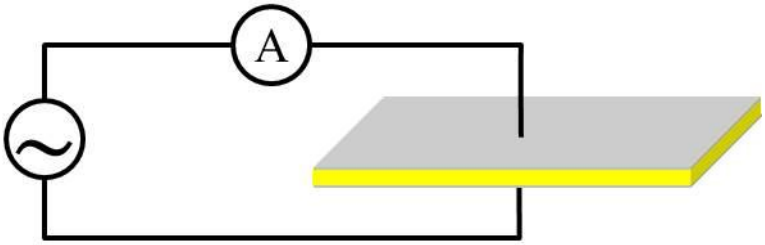


Figure 9. Circuit schematics for measuring the resonant and antiresonant frequencies.

B.2	Measure the AC current I through the PZT plate as a function of the signal frequency f. Draw the I-f curve and find the resonant frequency f_r and the antiresonant frequency f_a. Calculate the piezoelectric coefficient d accordingly.	3.5pts

Instructions:

(1) Connect the signal generator, the DMM and the PZT plate according to Figure 9. **Note that the PZT plate should be clamped at the middle point along the length direction using the Kelvin clip,** since the middle point is the node under the free boundary condition.

(2) The amplitude of the output signal changes little with frequency if the "Amplitude" knob of the signal generator remains untouched. However, the DMM may not respond to signals with a frequency above 40 kHz. Data above 40 kHz are not required.

(3) **Attention: keep the clamped PZT plate intact during the experiment. Slight movement may cause drifting of the current reading.**

(4) Attention: as mentioned in A.1, you are supposed to perform the measurement at ambient temperature. Avoid the direct warming up of the plate by your hand.

(5) Attention: if you do not hear any high pitch sound from your testing plate after sweeping frequency and you are confident with your measurement setup, you should contact the organizer as your instruments might be faulty.

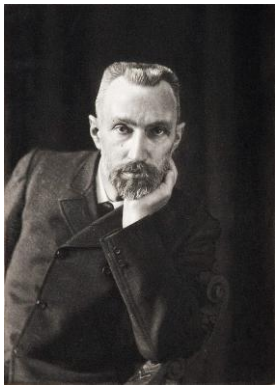
Experiment C

The Curie temperature of the PZT plate [4.0pts]

Most insulators have a dielectric susceptibility which is insensitive to the temperature variation. However, the static relative permittivity of the PZT ceramics changes with temperature according to the relation

$$\varepsilon_r = A + \frac{B}{T - T_c}, \text{ where } T > T_c.$$

Pierre Curie



Here A and B are constants independent of temperature. This relation is known as the **Curie-Weiss law**. Parameters B and T_c are called the Curie constant and the Curie temperature respectively, named after **Pierre Curie**.

A phase transition occurs at the transition temperature, T_c. Above T_c, the substance is in the paraelectric phase, in which the elementary dipoles of the various unit cells in the crystal are oriented randomly. Below T_c, the elementary dipoles interact with each other, giving rise to an internal field, which lines up the dipoles. A spontaneous polarization appears in the absence of an applied electric field. The relative permittivity below T_c is given as

$$\varepsilon_r = 1 + \frac{B}{2(T_c - T)}, \text{ where } T < T_c.$$

From **Experiment A** we know that $\varepsilon_r >> 1$. We can ignore the constant term to obtain an approximation of the static relative permittivity as

$$\varepsilon_r = \frac{B}{2(T_c - T)}, \text{ where } T < T_c.$$

Therefore capacitance of the PZT plate obtained from A.1 could vary with temperature as well. Since the Curie temperature of the PZT plate is much higher than the boiling temperature of water, we will estimate its Curie temperature by linear extrapolation.

C.1	Now measure the capacitance of the PZT plate at various temperatures and record the data.	1.5pts

Instructions:

(1) Use the cable with two banana plugs and two crocodile clips to connect the PZT plate

to the DMM. Pay attention to the polarity of the two crocodile clips when clamping the PZT plate. **Do not use the Kelvin clip because its ABS plastics will soften when heated.**

(2) Put the PZT plate inside the plastic bag. Use the plastic clip to clamp the cable and the bag. **Warning: one side of the bag is open. Do not tear the other side!**

(3) Pour the 1.5 L bottled water into the water bath. Immerse the plastic bag in the water. Use the pebble to keep the plastic bag in the water.

(4) Turn on the water bath and set the target temperature. **Warning: it is strictly prohibited to turn on the machine if the heating unit is not covered by water.**

(5) Warning: be very careful with the hot water in the water bath. Remember that water at temperatures higher than 50℃ can cause burns. For safety please do not set the temperature higher than 90℃.

(6) The temperature will rise slowly. Please record the capacitance C of the PZT plate at different temperatures.

(7) Turn off the water bath and unplug the power cord from the power socket when the measurement is finished.

(8) Hint: to speed up the experiment, you can choose to set the target temperature to 90℃ and record the capacitance as the temperate rises.

C.2	Analyze the data, draw a proper plot and calculate the Curie temperature accordingly.	2.5pts

Experiment D

Application: measuring the speed of sound in aluminum [6.5pts]

In solids, sound can be transmitted as both longitudinal waves and transverse waves. The medium movements responsible for the two types of waves are illustrated below.

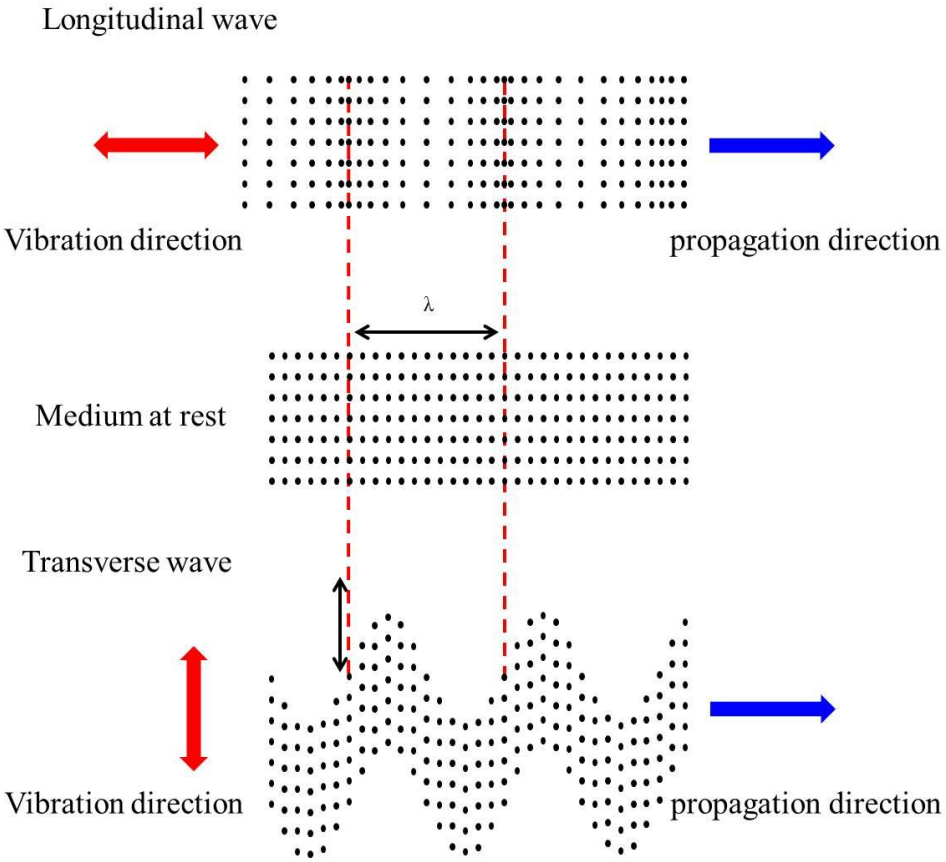


Figure 10. Longitudinal and transverse waves in solids.

In longitudinal waves, oscillation occurs in the longitudinal direction or the direction of wave propagation. Longitudinal waves can be generated in liquids, as well as in solids because the energy travels through the atomic structure by a series of compression and expansion movements.

In transverse waves, displacement of the medium is perpendicular to the direction of wave propagation. Transverse waves require a solid medium for propagation, and cannot effectively propagate in liquids or gasses.

In this experiment, we will measure the longitudinal and transverse wave velocities of sound in an aluminum rod. We will first measure the transverse wave velocity, and then the longitudinal wave velocity.

D.1	Assume that the length of the aluminum rod is L and the wave velocity is u. Under the free boundary condition, derive the equation for the frequencies f_n of the standing (resonant) waves along the long rod. Then derive the equation for the wave velocity u from f_n.	0.6pts

Now, we use a PZT plate as a transducer to produce sound waves in the aluminum rod, and use another PZT plate as a vibration sensor to detect the reflected sound waves.

First we measure the transverse wave velocity. As discussed in **Experiment B**, the vibration along the length direction is dominant. We position the transducer and the sensor to one end of the rod, as illustrated in Figure 11. The vibration of the transducer will propagate into the rod via friction, forming transverse waves.

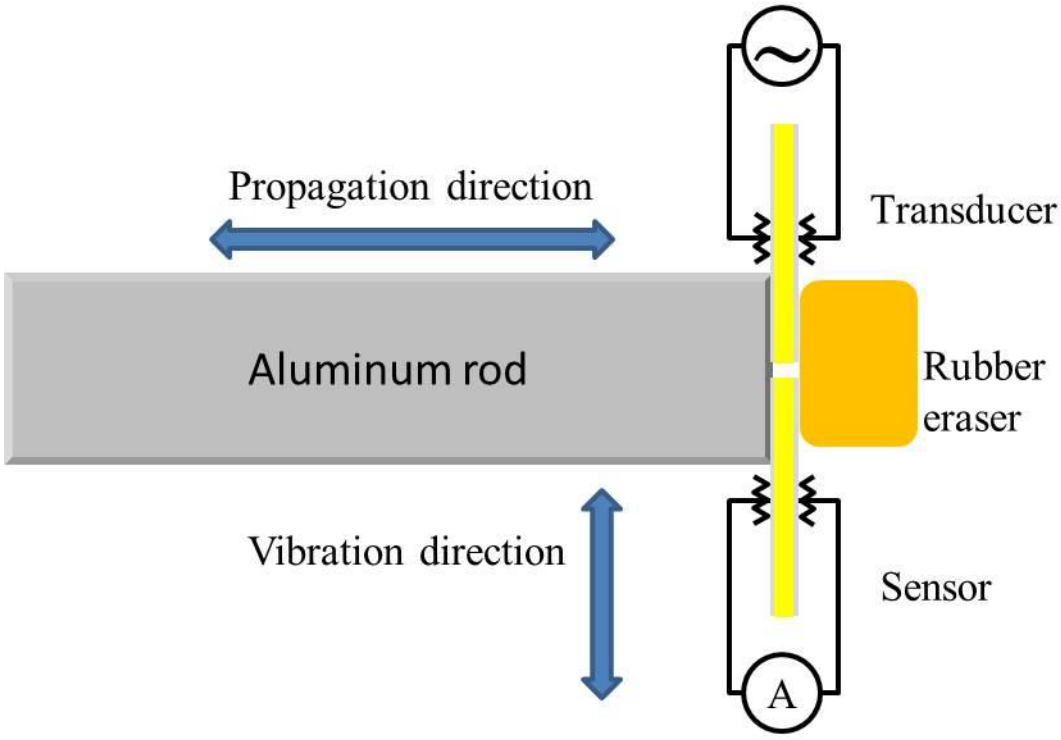


Figure 11. Illustration of the setup for measuring the transverse wave velocity (top view).

D.2	Use the steel tape measure to read the length L of the aluminum rod. Please repeat the measurement several times and calculate the mean and the standard error. While changing the frequency of the sound waves produced by the transducer, record the peak values monitored by the sensor. Draw a spectrum containing all measured resonant peaks, similar to that shown in Figure 12.	1.6pts

Instructions:

(1) Following steps 2 to 4 to arrange the experimental setup as illustrated in Figure 11. Note that the plastic box is specifically designed to accommodate the setup.
(2) Place the aluminum rod in the long trough of the plastic box, and use the eraser to press the PZT plates against one end of the aluminum rod, as illustrated in Figure 11. The two plates should not touch each other.
(3) Put the spring on the other end of the trough to gently push the rod against the PZT plates.
(4) Connect one PZT plate to the signal generator as the transducer. Connect the other PZT plate to the DMM as the sensor. Pay attention to the polarity when using the crocodile clips.
(5) The PZT plate is fragile and there is no extra supply.
(6) Use earplugs if you are annoyed by the high pitch sound.
(7) It is recommended that you sweep the frequency in the range between 0 and 40 kHz.

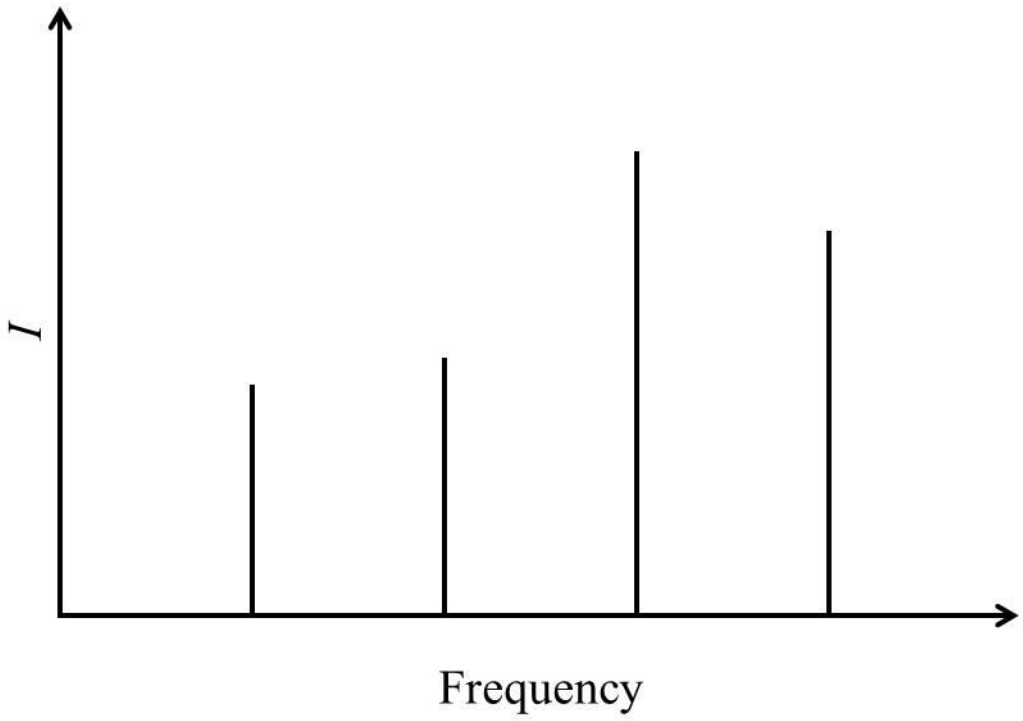


Figure 12. Exemplary spectrum showing all resonant peaks.

D.3	Identify the resonant peaks likely resulting from the transverse waves. Calculate the transverse wave velocity accordingly and carry out the error analysis. Attention: there might be irrelevant peaks caused by imperfection of the experimental setup, e.g., imperfect free boundary condition. You need to	1.4pts

	make a judgement and ignore the irrelevant peaks during your analysis.	

By changing the contact style between the PZT plates and the rod, we can also measure the longitudinal wave velocity in the rod. Attach the transducer and the sensor to the rod as illustrated in Figure 13. Vibration along the length direction of the transducer will propagate into the rod via compression, forming longitudinal waves.

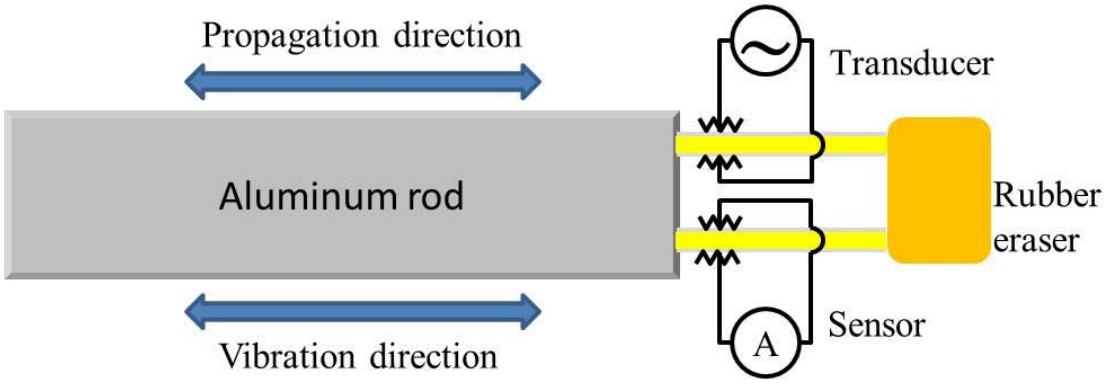


Figure 13. Illustration of the setup for measuring the longitudinal wave velocity.

As discussed in **Experiment B**, although vibration along the length direction is dominant, vibrations along other directions also exist, which could produce sound waves in the rod and introduce extra resonant peaks in the spectrum as well. Note that vibrations along other directions only introduce transverse waves. Therefore the extra resonant peaks will not coincide with the peaks caused by the longitudinal standing waves.

D.4	While changing the frequency of the sound waves produced by the transducer, record the peak values monitored by the sensor. Draw a spectrum containing all measured resonant peaks, similar to that shown in Figure 12.	1.5pts

Instructions:

(1) Following steps 2 to 5 to arrange the experimental setup as illustrated in Figure 13. Note that the plastic box is specifically designed to accommodate the setup.
(2) Align two PZT plates in the two slim slots, respectively, on one end of the plastic box.
(3) With the long aluminum rod in the trough, use the eraser as a cushion to press the PZT plates against one end of the rod.
(4) Insert the spring on the other end to gently push the rod against the PZT plates.
(5) Connect one PZT plate to the signal generator as the transducer. Connect the other PZT plate to the DMM as the sensor. Note the polarity when using the crocodile clips.
(6) **Attention: the style of contact between the PZT plates and the rod is crucial.**

Please have the edges of the PZT plates fully in touch with the side of the rod, so as to avoid point or partial contact.

(7) **Attention: if you find too many resonant peaks in the spectrum, you can try to reduce the signal amplitude from the signal generator and/or slightly release the pressure between the PZT plates and the rod to reduce the production of transverse waves. You can also try to remove the spring on the other end if the contact between the PZT plates and the rod is still decent without the spring.**

(8) It is recommended that you sweep the frequency between 0 and 40 kHz.

D.5	Compare with the result in D.2, identify the resonant peaks caused by the transverse waves. Select the resonant peaks resulting from the longitudinal waves and calculate the longitudinal wave velocity accordingly. Carry out the error analysis.	1.4pts

Experiment E

Application: locating a defect in an aluminum rod [2.0pts]

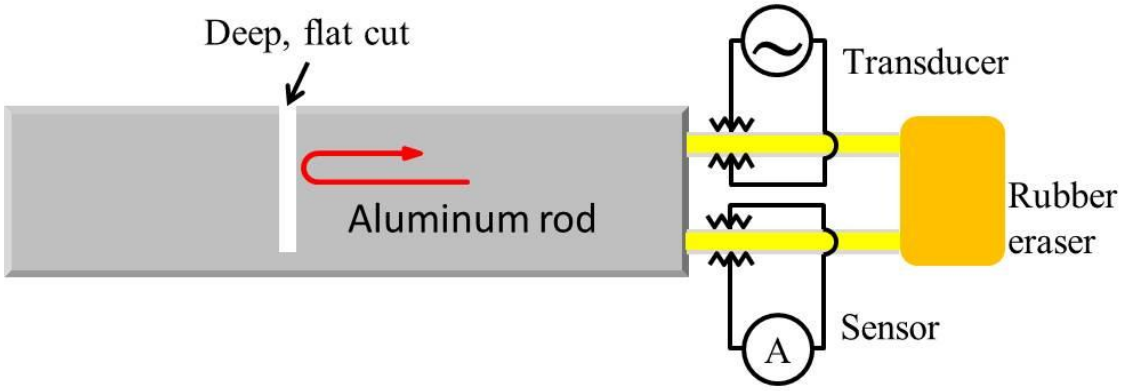


Figure 14. Illustration of the setup for locating a defect in an aluminum rod.

After knowing the longitudinal wave velocity from **Experiment D**, here we attempt to locate an artificially-engineered defect, a deep cut, along the long aluminum rod.

Sound waves through solid materials have been used to detect hidden cracks, voids, and other internal discontinuities in metals, composites, plastics, and ceramics. The industrial ultrasonic testing is done by repeatedly generating a several megahertz short pulse with an amplitude up to several hundred volts to drive the transducer, and then by amplifying the received signal to see if there is any reflection from a flaw. This is technically too complicated for our current setup. Instead, as a simple demonstration, we use the resonant method again to detect a deep, flat cut in the rod. Different from the case in **Experiment D**, sound waves will be bounced back at the spot of the deep cut instead of the far end of the rod. Therefore arrangement of the resonant peaks in the spectrum will be different from that in **Experiment D**, which can be the criterion for locating the spot of the deep cut.

The detection can be done using longitudinal waves.

E.1	While changing the frequency of the sound waves produced by the transducer, record the peak values monitored by the sensor. Draw a spectrum containing all measured resonant peaks, similar to that shown in Figure 12.	1.2pts

E.2	In the measured spectrum, identify the resonant peaks corresponding to the existence of the deep cut. Estimate the distance from the spot of the cut to the end of the rod that is in contact with the PZT plates.	0.8pts