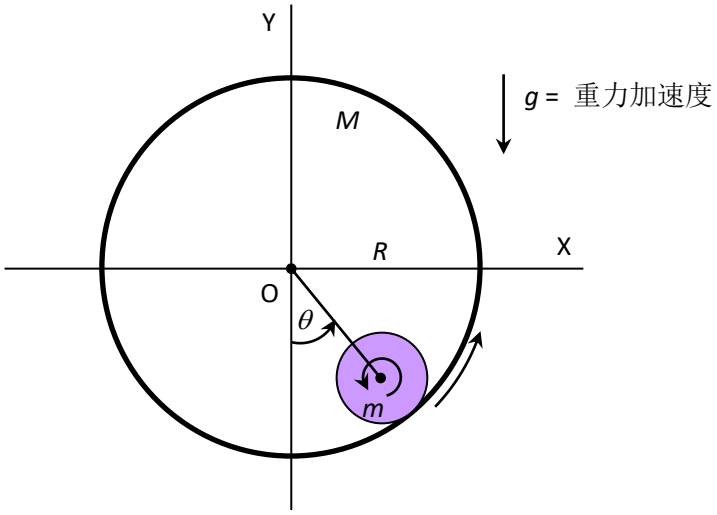
滚动圆柱体

有一质量为 M 的薄空心圆筒，其内表面为粗糙表面。圆筒内表面的半径为 R 。圆筒可以绕其沿水平方向的中轴 OZ 转动。圆筒的中轴是固定的。Z 轴的方向垂直于纸面。在圆筒内部，另有一个质量为 m 、半径为 r 的较小的实心圆柱体。较小的实心圆柱体可以绕其自身的中轴在薄空心圆筒 M 的内表面无滑动地滚动（在最后一问即问题 1.8 中除外）。实心圆柱体的中轴与 OZ 轴平行。



1.1) 在 $t=0$ 的时刻，圆筒 M 由静止状态开始转动，而此时圆柱体 m 处于最低位置。经过 t 时间后圆柱体 m 的质心到达角位置 θ，而在同一段时间内圆筒 M 转动了 ϕ 弧度。试求在此段时间内圆柱体 m 绕其自身的中轴转动的角度（用 ψ 弧度表示）。结果用 θ 、ϕ 、R 和 r 表达。 （0.8 分）

1.2) 试求圆柱体 m 绕其自身的中轴转动的角加速度 $\frac{d^2}{dt^2}\psi$ ，结果用 R 、r 及 θ 和 ϕ 对时间的某阶导数表达。 （0.2 分）

1.3) 导出关于圆柱体 m 质心的角加速度 $\frac{d^2}{dt^2}\theta$ 的方程。方程用 $m, g, R, r, \theta, \frac{d^2}{dt^2}\phi$ 和 I_{CM} 表达，其中 I_{CM} 是圆柱体 m 相对其自身中轴的转动惯量。 （1.8 分）

1.4) 试求在圆筒 M 以恒定的角速度转动的情况下，圆柱体 m 作小振幅振动的周期。结果只能用 R 、r 和 g 表达。 （1.3 分）

1.5) 试求问题 1.4 中圆柱体 m 平衡位置所对应的 θ 角。 （0.2 分）

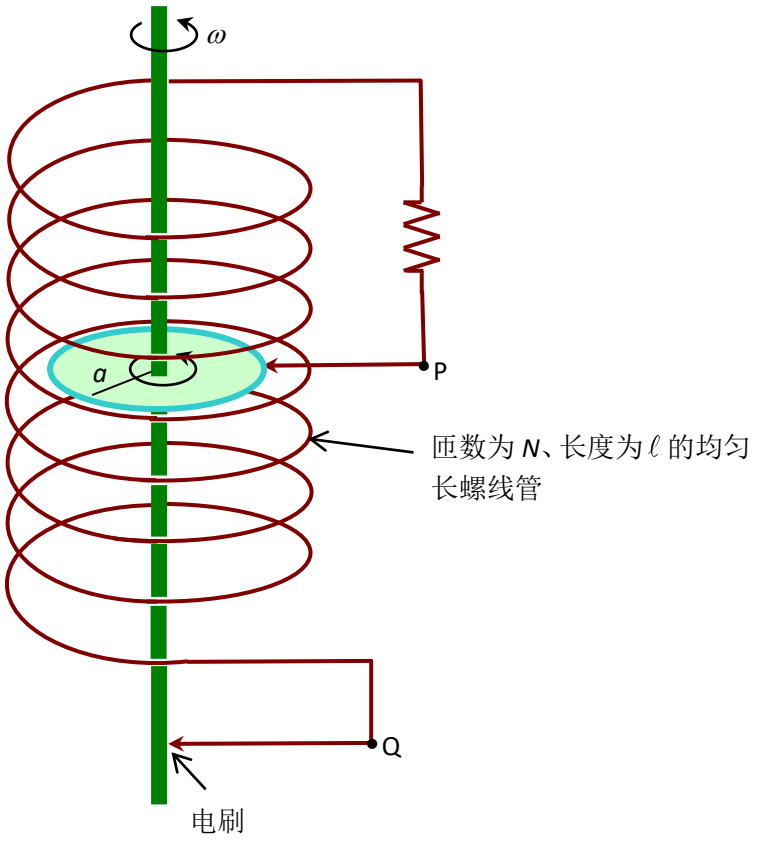
1.6) 试求在圆筒 M 以恒定的角加速度 α 转动的情况下，圆柱体 m 的平衡位置。结果用 R 、g 和 α 表达。（0.7 分）

1.7) 现在假设圆筒 M 可以不受约束地绕其中轴 OZ 自由转动或往复振动，而圆柱体 m 作小振幅振动。在振动的过程中，圆柱体 m 相对圆筒 M 的内表面的运动为纯滚动。试求圆柱体 m 的振动周期。（2.5 分）

1.8) 考虑以下情形：开始时圆筒 M 以固定的角速度 Ω 转动，圆柱体 m 在问题 1.5 中所确定的平衡位置处，绕其自身的中轴转动（滚动）。此时圆柱体 m 质心的位置保持不变。然后圆筒 M 突然停止转动。假设圆柱体 m 与圆筒 M 内表面之间有足够大的摩擦系数，使得圆柱体 m 在圆筒 M 停止转动之后，经过一个短暂的非纯滚动过程，然后开始在圆筒 M 内表面作纯滚动。要使圆柱体 m 能滚到圆筒 M 内表面最高处，则角速度 Ω 的最小取值应为多少？（2.5 分）

自激式发电机

如图所示，一个半径为 a 的金属圆盘安装在一个电感为 L 的长螺线管的轴上。金属圆盘在螺线管中以恒定的加速度 ω 转动，而螺线管的两端则通过两个电刷联接到转动的金属圆盘上。整个电路的总电阻为 R 。磁场的小扰动可以导致 P、Q 两端之间的电动势的增长。



2.1) 采用 L、R 和 PQ 间的感生电动势 (E) 为参量，写出确定电路中电流 $i(t)$ 随时间变化的微分方程。（1.0 分）

2.2) 试用 i、N、ℓ 和真空磁导率 μ_0 表达螺线管中的磁感应强度 (B)。金属圆盘和轴上的电流产生的磁场可以忽略。（1.5 分）

2.3) 试用 μ_0、N、a、ℓ、i 和角速度 ω 表达感生电动势 (E)。（2.0 分）

2.4) 求解问题 2.1) 中得到的微分方程，用初始电流 $i(0)$ 等参量表达任意时刻 t 电路中的电流。（1.5 分）

2.5) 试求能使电流不断增长的角速度的最小值。结果用 R, μ_0, N, a 和 ℓ 表达。（2.0 分）

2.6) 需要在轴上施加一个力矩才能使角速度 ω 保持某一稳定的值。试求在 t 时刻此力矩的值。（2.0 分）

莱顿弗罗斯特现象

一个置于炽热平板上的（半球形）液滴与平板间会有一层很薄的蒸汽将其与平板隔开，本题的任务就是要估计这样一个液滴的寿命。

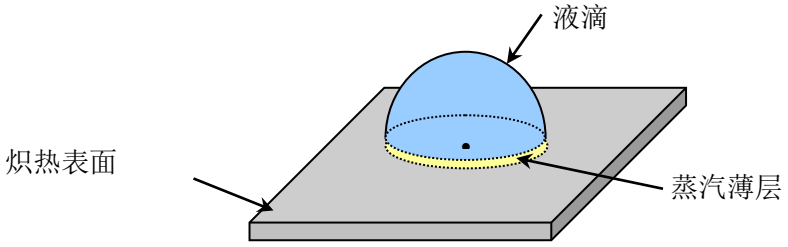


图 1

假定液滴下的气流满足层流条件，气体可看成是热导率为K，粘滞系数为η的牛顿流体。液体的比汽化潜热为ℓ。对牛顿流体，切应变$\frac{F}{A}=\eta\times$切变速率$\frac{dv}{dz}$，这里v是流速，z垂直于流速方向，F 沿面积 A 的切线方向。

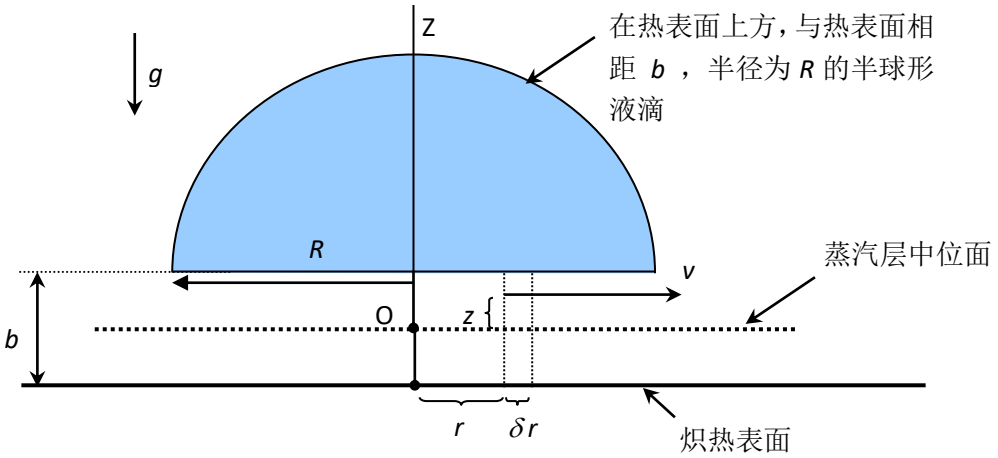


图 2

v是中位面上方高度为z的位置处蒸汽沿径向的速度。蒸汽层中的压强P一定是越靠近中心 O 越大。这将会使蒸汽从内向外流并使得液滴可以克服重力悬浮。由热平衡和力学平衡所决定的蒸汽层的厚度为b。

对牛顿蒸汽流近似有：

$$\frac{d}{dz}v = \frac{z}{\eta}\frac{d}{dr}P$$

3.1) 证明：$v(z) = \frac{z^2}{2\eta}\frac{d}{dr}P + C$

这里，C 是一个任意的积分常数。 (0.5 分)

3.2) 参照图 2，利用边界条件 $z = \pm\frac{b}{2}$ 时 $v = 0$，将 C 用 $\eta, \frac{d}{dr}P$，和 b 表示出来。 (0.5 分)

3.3) 计算流过由 r 定义的圆柱体表面的蒸汽的体积流量。（提示：圆柱体在液滴之下，其半径为 r 高度为 b ） (1.0 分)

3.4) 假定密度为 ρ_V 的蒸汽的产生速率是由炽热表面流向液滴的热流决定的，请给出压强 $P(r)$ 的表达式。用 P_a 表示大气压强，ΔT 表示炽热表面与液滴间的温度差。推导时假定系统处于稳定状态。 (2.0 分)

3.5) 液滴的上下表面压强不同，会产生一个净力。由此力等于液滴的重量计算出 b 的值。液滴的密度为 ρ_0。 (2.0 分)

3.6) 现在请回答，总蒸发速率是多少？ (2.0 分)

3.7) 假定液滴始终保持半球形状，请问其寿命有多长？ (2.0 分)

****

试题 1：地磁场水平分量的测量

本实验的目的是利用一个圆柱形小磁铁棒的小幅振动来测定地磁场磁感应强度的水平分量 B_{H}。在本实验中，小磁棒在由地磁场和正方形线圈产生的磁场叠加而成的静磁场中振动。

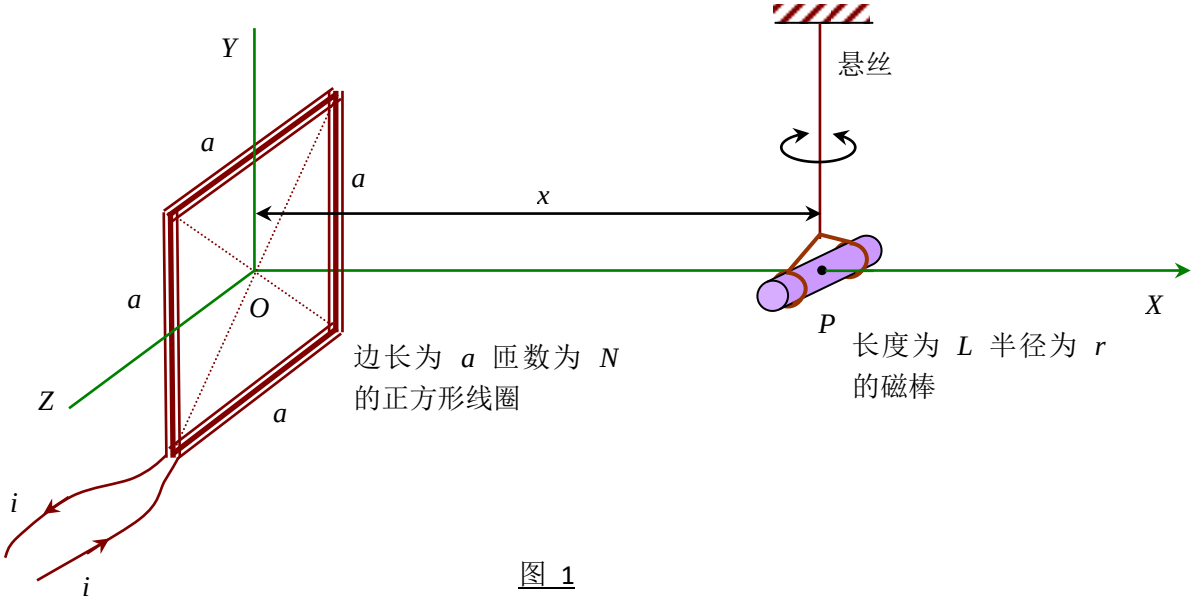


图 1

本实验试题包括三个部分。在第一部分中需推导出用于第三部分的公式。

装置

实验装置如图 2 所示，其中：

1. 一个电阻为$5.2\pm0.2\ \Omega$，匝数为 130 的正方形线圈；
2. 一个质量为15.0 ± 0.2 g，拴有尼龙悬线的圆柱形小磁铁棒；
3. 一个电压表（仅用于测量线圈两端的电位差）；
4. 一个电源（为避免电源本身磁场的影响，电源被放置在实验台下面）；
5. 一个木质支架；
6. 一块秒表；
7. 一把直尺；
8. 一个量角器；
9. 铺在平台上的白标记纸（可在其上写、画）；
10. 胶泥块；
11. 绘图纸；
12. 电线。

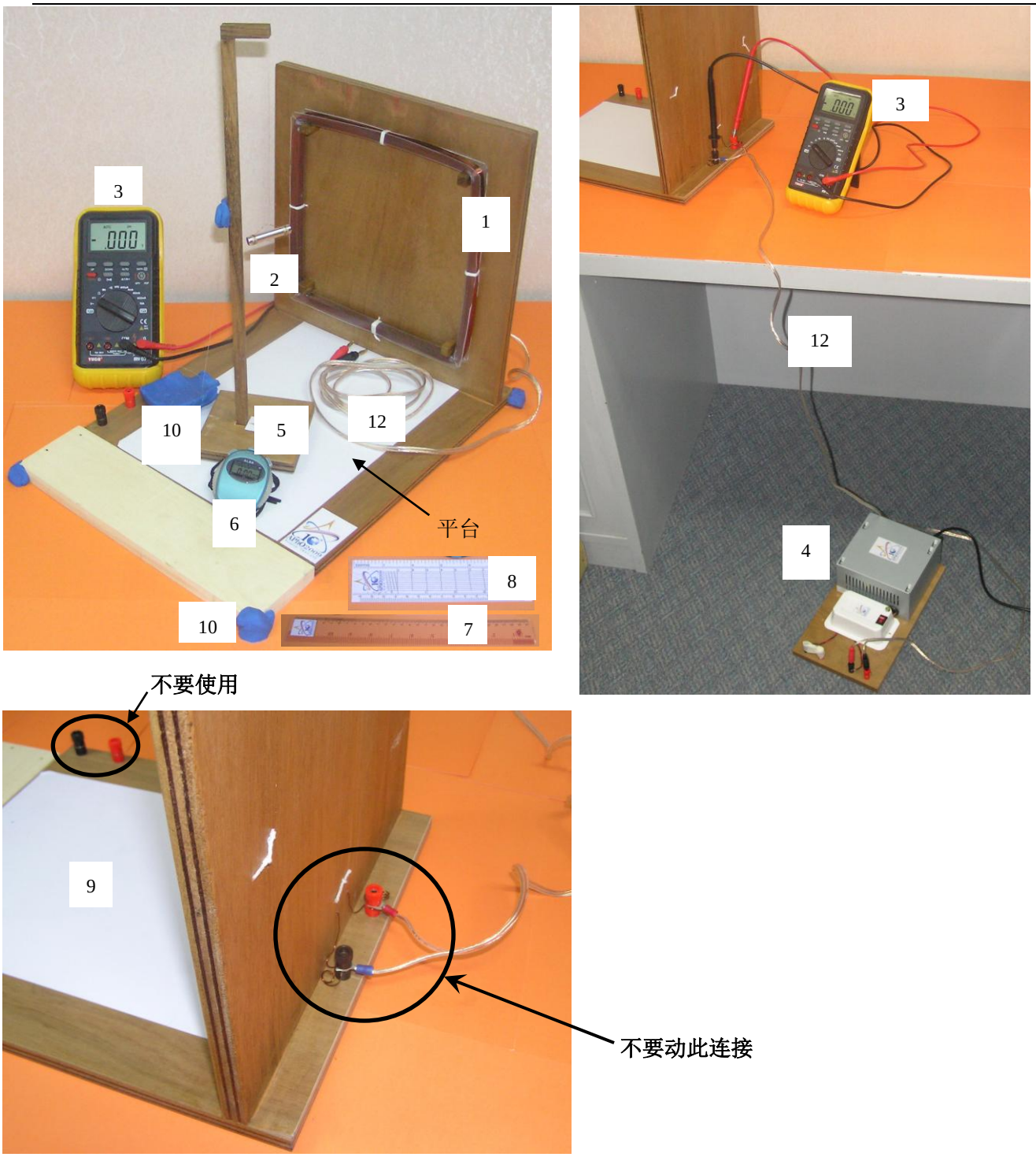


图 2

警告

万用表仅作为电压表用于测量线圈两端的电位差。将万用表置于其它工作模式可能导致电源的损坏。

第一部分

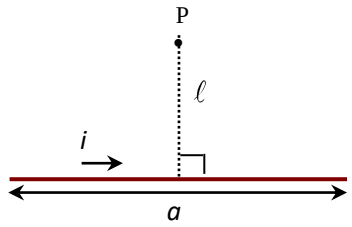


图 3

[1 point]

这里给出一个直电流元 ia 在其中垂线上距离 ℓ 处产生的磁场的磁感应强度 B_{P} 的表达式

$$B_{\mathrm{P}} = \frac{\mu_0 i}{2\pi\ell}\frac{(a/2)}{\sqrt{\ell^2+\left(\frac{a}{2}\right)^2}} \qquad \text{..... (i)}$$

其中 $\mu_0 = 4\pi\times10^{-7}$ 亨/米，为真空磁导率。

利用上式证明图 1 中正方形线圈在 P 点产生的磁场的磁感应强度为：

$$B_{px} = \left(\frac{\mu_0 a^2 iN}{2\pi}\right)\left[\frac{1}{\left(x^2+\left(\frac{a}{2}\right)^2\right)\sqrt{x^2+2\left(\frac{a}{2}\right)^2}}\right] \qquad \text{..... (ii)}$$

这里同样给出磁铁棒在磁感应强度为 B 的磁场中作小幅振动的周期的表达式：

$$T = 2\pi\sqrt{\frac{I}{mB}} \qquad \text{..... (iii)}$$

其中 m 为磁铁棒的磁矩，磁铁棒的质量为 M 。I 为磁铁棒相对于通过其质心的转轴的转动惯量

$$I = M\left(\frac{L^2}{12}+\frac{r^2}{4}\right) \qquad \text{..... (iv)}$$

第二部分

[0.8 分]

在第三部分的实验测量中，磁铁棒位置应按图 1 示意的方式悬挂。如果悬线长度过短，则悬线的扭矩对磁铁棒振动的影响不能忽略。作适当的测量（譬如，测量仅有地磁场时磁铁棒的振动），用得到的测量结果来证明悬线的扭矩对后面实验的影响可被忽略。这一部分不用画图。

第三部分

在下面的实验步骤（a、b 和 c）中，磁铁棒位置应按图 1 示意的方式悬挂。测量并记下磁铁棒中心到平台上表面的距离。 [0.2 分]

a) **线圈磁场与地磁场水平分量方向相同的情形** [5 分]

警告
将线圈连接到电源后需至少等候 5 分钟再开始测量。

使线圈产生的磁场与地磁场的方向一致。测量不同的总磁场强度下磁铁的振动周期。作一条直线图，根据此直线的参数计算出 B_{H} 和磁矩 m 的值，并估算它们的误差。

b) **仅有地磁场的情形** [1 分]

用步骤（a）得到的小磁棒磁矩值 m 和第二部分得到的不加线圈磁场时小磁棒的振动周期，再次计算地磁场磁感应强度水平分量 B_{H} 的值，并估算其误差。

c) **线圈磁场与地磁场水平分量反向的情形** [2 分]

改变**电源端**接线柱处的接线极性，沿 X 轴寻找地磁场与反向的线圈磁场在 X 轴方向抵消的位置 x_0 。再次根据 x_0 计算 B_{H} 的值，并估算其误差。

****

试题 2：充水容器的振动

本实验的目的是采用非破坏性方法测量一个充满水的铝质容器的侧壁厚度 t。实验中所用的铝质容器由一个圆柱形空筒和两个端盖组成。圆柱形空筒的长度为 L，外半径为 R。容器的总长度为 h。两个端盖的厚度均为 *0.60 cm*（见图 1）。端盖厚度的误差可以忽略。**请使用 *cm* 作为长度的单位，*g* 作为质量的单位。**

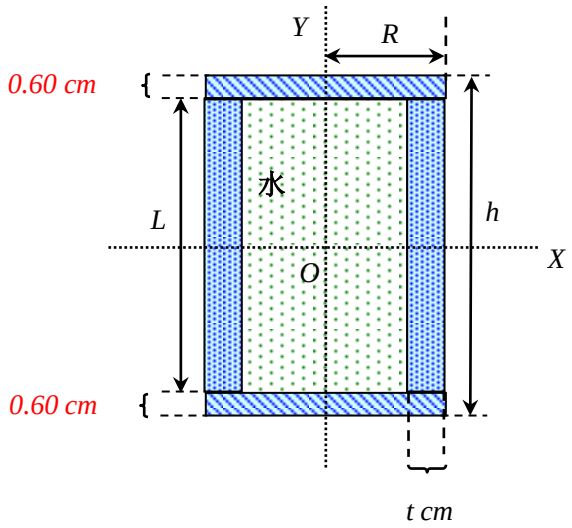


图 1

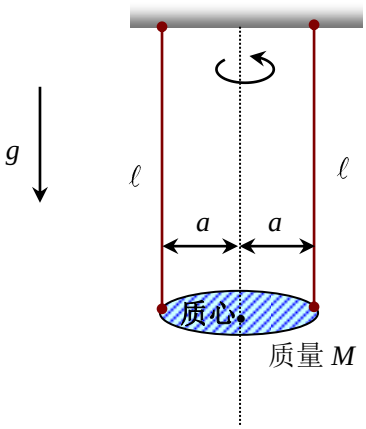


图 2

图 2 显示的是一个质量为 M 的物体的所谓双丝悬挂。双丝的长度均为 ℓ。M 作小幅角振动的周期 T 由下式给出：

$$T = 2\pi\sqrt{\frac{\ell}{g}\cdot\frac{I}{Ma^2}} \qquad \text{..... (i)}$$

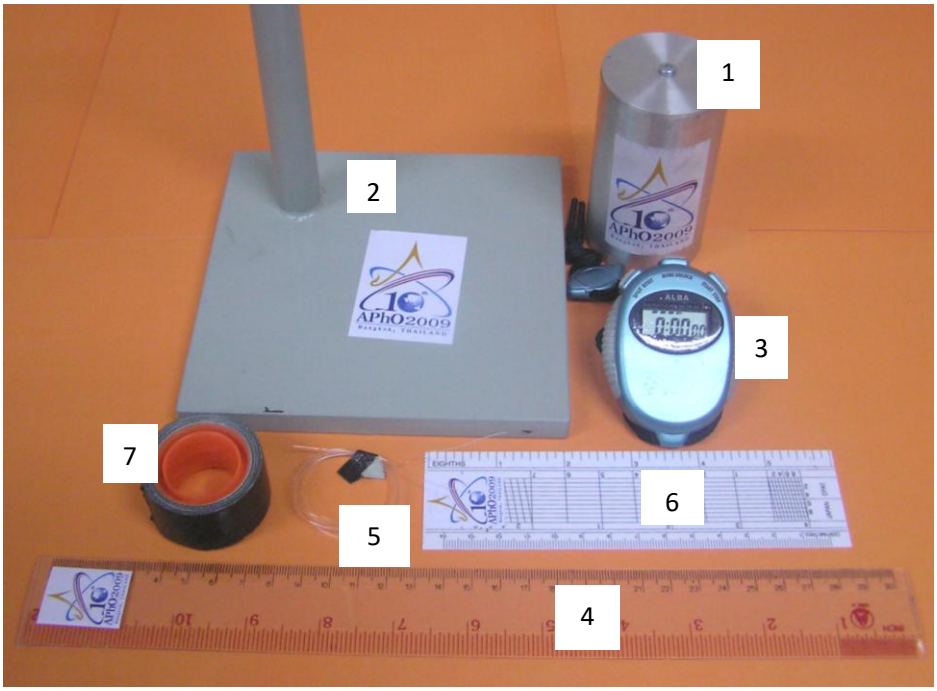
其中 I 为物体相对通过其质心的竖直轴的转动惯量，g 为在曼谷的重力加速度 $\left(g = 978\ \text{cm s}^{-2}\right)$。

本实验试题包括两个部分，第一部分为公式推导，第二部分为实验操作。

装置

每套实验装置包括：

1. 一个充满水的铝质容器；
2. 一个支架；
3. 一块秒表；
4. 一把直尺；
5. 一根尼龙悬丝；
6. 一个量角器；
7. 一卷胶带
8. 一把小刀（图中未画出）



第一部分

[2 分]

首先请**推导出**下列各量的表达式，结果用 R、L、t 和铝的密度 ρ 表达：[见图 1]

i) 圆柱形空筒的质量 (m_1)；

ii) 每个端盖的质量 (m_2)；

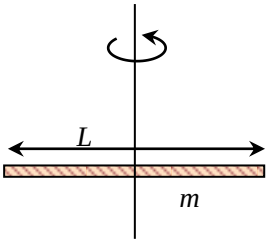
iii) 充满容器的水的质量 (m_3)；

iv) 容器充满水后的总质量 (M)；

v) 充满水的容器相对 Y 轴（见图 1）的有效转动惯量 I_y，假设水为**理想流体**。

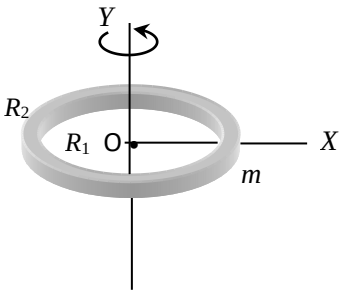
然后测量 R、h、L，并代入所导出的各表达式中，进而得到以上各量的**以 t 为参数的数值表达式**。已知铝的密度为 $\rho = 2.70$ g/cm^3，水的密度为 1.00 g/cm^3。

提示：



$$I = m\frac{L^2}{12}$$

长度为L的细杆



$$I_y = \frac{1}{2}m\left(R_2^2 + R_1^2\right),$$
$$I_x = \frac{1}{4}m\left(R_2^2 + R_1^2\right)$$

内半径为R_1、外半径为R_2的圆筒

图 3

第二部分

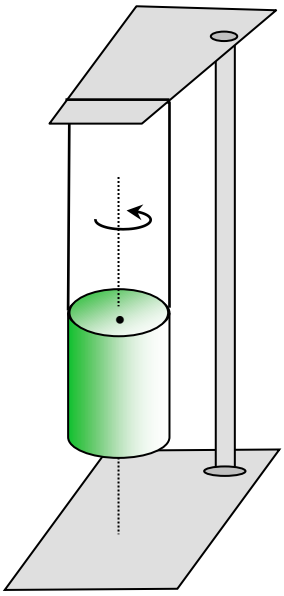


图 4

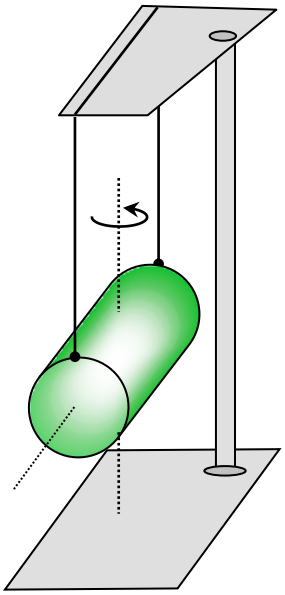


图 5

a) **相对于对称轴的角振动**

[4 分]

采用图 4 所示的悬挂方法，对一固定的悬丝长度ℓ，**精确地**测量小幅振动的周期T_y。然后计算圆筒壁厚度(t)的值。

估算厚度(t)的测量误差 Δt。

用得到的 t 值计算 m_1、m_2、m_3 和 M 的值。

b) **相对于过容器中心且垂直于对称轴的转轴的角振动**

[2.8 分]

改用图 5 所示的悬挂方法，进行与步骤（a）类似的测量。

用这里测得的振动周期值和步骤（a）所得 t, m_1, m_3, M 中合适的值，计算充满水的容器相对 X 轴（见图 2 和图 5）的有效转动惯量 $I_{\mathrm{x}}^{\mathrm{Exp}}$。

假设容器中的水全部都与容器同步振动。采用步骤（a）中得到的 t 值，计算转动惯量的理论值 $I_{\mathrm{x}}^{\mathrm{Theo}}$。

c) **转动惯量的实验值与理论值的比较**

[1.2 分]

$I_{\mathrm{x}}^{\mathrm{Theo}}$ 和 $I_{\mathrm{x}}^{\mathrm{Exp}}$ 的差 $\left(\Delta I_{\mathrm{x}}\right)$ 是多少？

你认为这个差在统计上有意义吗？

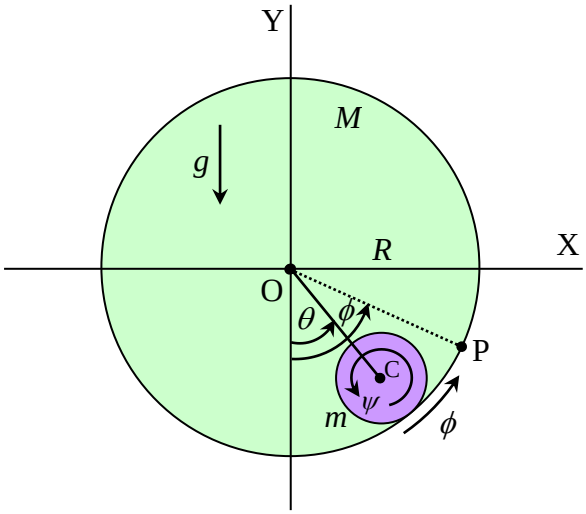
作为一个简化的模型，假设在步骤（b）中，仅仅容器两端各有一段圆柱形水柱发生运动，且运动与容器同步。请估算参与振动的水的质量占容器中水的总质量的百分比。

提示：

$$I_{\mathrm{x}}^{\mathrm{Theo}} = m_1\left[\frac{L^2}{12}+\frac{R^2+\left(R-t\right)^2}{4}\right]+2m_2\left[\frac{\left(0.6\ \mathrm{cm}\right)^2}{12}+\frac{R^2}{4}+\left(\frac{L}{2}+\frac{0.6\,\mathrm{cm}}{2}\right)^2\right]+m_3\left[\frac{L^2}{12}+\frac{\left(R-t\right)^2}{4}\right]$$

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Rolling Cylinders



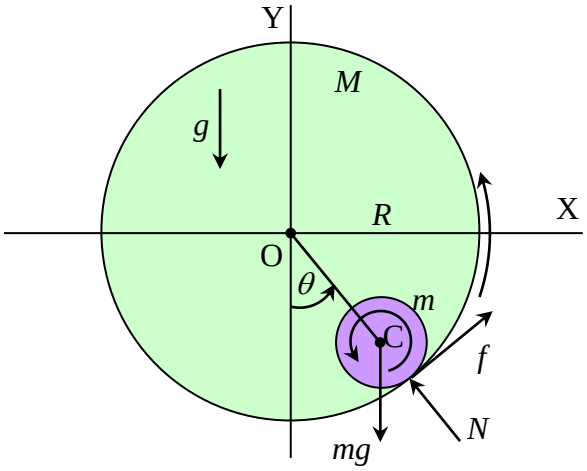
1.1) The point P which is fixed on the surface of M was at the position directly below O at time $t=0$. Hence m must have rolled through an angle $\frac{\phi R-\theta R}{r}$ radians relative to surface of M in time t during which the line OC has also turned anti-clockwise through an angle θ. Therefore the total angular displacement of m about its centre of mass relative to any fixed reference line in time t is

$$\psi = \frac{\phi R-\theta R}{r}+\theta = \frac{R}{r}\phi-\left(\frac{R-r}{r}\right)\theta \qquad \text{.....(i)} \qquad \text{(0.8 point)}$$

1.2) By differentiating the equation (i) twice with respect to time, we get

$$\frac{d^2}{dt^2}\psi = \frac{R}{r}\frac{d^2}{dt^2}\phi-\left(\frac{R-r}{r}\right)\frac{d^2}{dt^2}\theta \qquad \text{.....(ii)} \qquad \text{(0.2 point)}$$

1.3)



The equations of motion of centre of mass of m are:

$$m(R-r)\frac{d^2}{dt^2}\theta = f - mg\sin\theta \quad \text{.....(iii)} \quad \text{(0.7 point)}$$

$$m\left(\frac{d}{dt}\theta\right)^2(R-r) = N - mg\cos\theta \quad \text{.....(iv)}$$

The equation for the rotation of m about its centre of mass is:

$$I_{\text{CM}}\frac{d^2}{dt^2}\psi = I_{\text{CM}}\left[\frac{R}{r}\frac{d^2}{dt^2}\phi - \left(\frac{R-r}{r}\right)\frac{d^2}{dt^2}\theta\right] = fr \quad \text{.....(v)} \quad \text{(0.7 point)}$$

where $I_{\text{CM}} = \frac{1}{2}mr^2$.

Equations (iii) and (v) yield:

$$\left(m + \frac{I_{\text{CM}}}{r^2}\right)(R-r)\frac{d^2}{dt^2}\theta = -mg\sin\theta + \frac{I_{\text{CM}}R}{r^2}\frac{d^2}{dt^2}\phi \quad \text{.....(vi)} \quad \text{(0.4 point)}$$

1.4) Here, $\frac{d^2}{dt^2}\phi = 0$, $\sin\theta \approx \theta$ and also $I_{\text{CM}} = \frac{1}{2}mr^2$, the equation (vi) is reduced to:

$$\frac{d^2}{dt^2}\theta = -\frac{2g}{3(R-r)}\theta \quad \text{.....(vii)} \quad \text{(0.8 point)}$$

This gives a period

$$T = 2\pi\sqrt{\frac{3(R-r)}{2g}} \quad \text{.....(viii)} \quad \text{(0.5 point)}$$

1.5) The equilibrium position of m in question 1.4) is $\theta = 0$. (0.2 point)

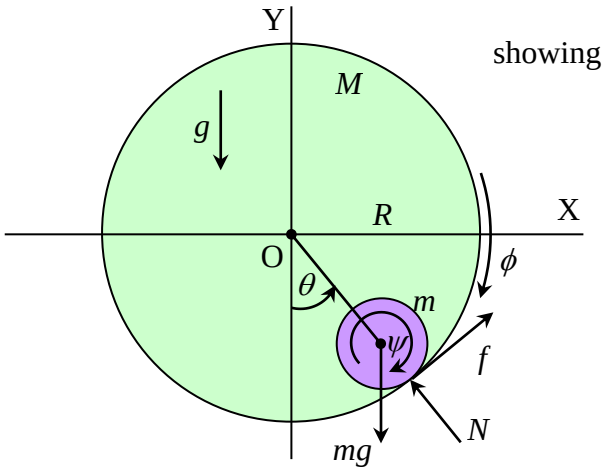
1.6) But the equilibrium position for the case where M is rotating with a constant angular acceleration α is by considering the equation (vi), namely,

$$\frac{3}{2}(R-r)\frac{d^2}{dt^2}\theta = -g\sin\theta + \frac{R}{2}\alpha \quad \text{.....(ix)} \quad \text{(0.4 point)}$$

Let θ_{eq} be the equilibrium position; this implies that m remains stationary at this position if it does not oscillate. Hence $\frac{d^2}{dt^2}\theta_{eq} = 0$, and (0.1 point)

$$\theta_{eq} = \arcsin\left(\frac{R\alpha}{2g}\right) \quad \text{……..(x)} \quad \text{(0.2 point)}$$

1.7)



showing forces acting on m

From the equation (i) we get, after changing the directions of ψ and ϕ,

$$\frac{d}{dt}\psi = \frac{R}{r}\frac{d}{dt}\phi + \left(\frac{R-r}{r}\right)\frac{d}{dt}\theta \quad \text{……..(xi)} \quad \text{(0.5 point)}$$

The equations of motion of m and M are:

$$\frac{1}{2}mr^2\frac{d^2}{dt^2}\psi = -fr \quad \text{……..(xii)} \quad \text{(0.3 point)}$$

$$MR^2\frac{d^2}{dt^2}\phi = +fR \quad \text{……..(xiii)} \quad \text{(0.3 point)}$$

Method 1: (Angular Momentum)

The effect of gravity on the system as a whole is to change its angular momentum:

$$\frac{d}{dt}\left[MR^2\frac{d}{dt}\phi + \frac{1}{2}mr^2\frac{d}{dt}\psi - m(R-r)^2\frac{d}{dt}\theta\right] = +mg(R-r)\sin\theta \quad \text{……..(xiv.1)} \quad \text{(0.4 point)}$$

Hence $$\frac{d^2}{dt^2}\phi = -\frac{m(R-r)}{(2M+m)R}\frac{d^2}{dt^2}\theta \quad \text{……..(xv.1)}$$

and $$\left(MR+\frac{1}{2}mr\right)R\frac{d^2}{dt^2}\phi - m(R-r)\left(R-\frac{3}{2}r\right) = mg(R-r)\sin\theta \quad \text{……..(xvi.1)} \quad \text{(0.4 point)}$$

Combining the last two equations:

$$\frac{d^2}{dt^2}\theta = -\frac{g}{(R-r)}\frac{(2M+m)}{(3M+m)}\sin\theta \qquad \text{……..(xvii.1)}$$

For a small-amplitude oscillation we put $\sin\theta \approx \theta$ and equation (xvii) is reduced to:

$$\frac{d^2}{dt^2}\theta = -\frac{g}{(R-r)}\frac{(2M+m)}{(3M+m)}\theta \qquad \text{……..(xviii.1)} \quad \text{(0.5 point)}$$

The period of this oscillation is, therefore,

$$T = 2\pi\sqrt{\left(\frac{R-r}{g}\right)\left(\frac{3M+m}{2M+m}\right)} \qquad \text{(0.1 point)}$$

Method 2: (Newton's law)

From Newton's law: $mg\sin\theta - f = ma$

$$mg\sin\theta - f = -m(R-r)\frac{d^2\theta}{dt^2} \qquad \text{……..(xiv.2)} \quad \text{(0.4 point)}$$

From equation (xiii): $f = MR\frac{d^2\phi}{dt^2}$

Substitute this into equation (xiv.2) we have

$$mg\sin\theta = MR\frac{d^2\phi}{dt^2} - m(R-r)\frac{d^2\theta}{dt^2} \qquad \text{……..(xv.2)}$$

From equations (xi) (xii) and (xiii), we then have

$$\frac{d^2\phi}{dt^2} = -\frac{m}{2M+m}\left(\frac{R-r}{R}\right)\frac{d^2\theta}{dt^2} \qquad \text{……..(xvi.2)} \quad \text{(0.4 point)}$$

Then (xv.2) becomes $mg\sin\theta = -\frac{Mm}{2M+m}(R-r)\frac{d^2\theta}{dt^2} - m(R-r)\frac{d^2\theta}{dt^2}$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{(R-r)}\frac{2M+m}{3M+m}\sin\theta \qquad \text{……..(xvii.2)}$$

For a small-amplitude oscillation we put $\sin\theta \approx \theta$ and equation (xvii) is reduced to:

$$\frac{d^2}{dt^2}\theta = -\frac{g}{(R-r)}\frac{(2M+m)}{(3M+m)}\theta \qquad \text{……..(xviii.2)} \quad \text{(0.5 point)}$$

The period of this oscillation is, therefore,

$$T = 2\pi\sqrt{\left(\frac{R-r}{g}\right)\left(\frac{3M+m}{2M+m}\right)} \qquad \text{(0.1 point)}$$

1.8) When M is made to rotate steadily at an angular velocity Ω the equation (vi) becomes

$$\frac{3}{2}(R-r)\frac{d^2}{dt^2}\theta = -g\sin\theta \qquad \text{……..(xix)}$$

which implies that m remains at $\theta = 0$ if m does not oscillate.

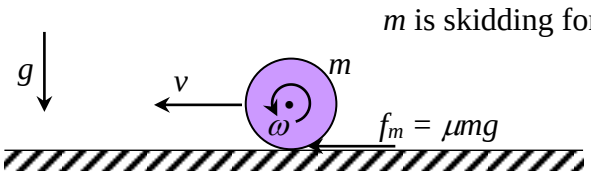
Hence the equation (i) is reduced to

$$\psi = \frac{R}{r}\phi$$

and $$\frac{d}{dt}\psi = \frac{R}{r}\frac{d\phi}{dt} = \frac{R}{r}\Omega \qquad \text{……..(xx)} \qquad \text{(0.2 point)}$$

This means that m is rotating at a constant angular velocity $\frac{R}{r}\Omega$ prior to the instant when M is stopped.

After that instant m will accelerate itself by way of frictional impulse. This acceleration process lasts for only a short time due to the high value of frictional coefficient (μ). To simplify the calculation we will take to lower surface of M to be flat.



$$m\frac{d}{dt}v = +f_m \qquad \text{……..(xxi)}$$

$$I_{CM}\frac{d}{dt}\omega = -f_m r, \quad I_{CM} = \frac{1}{2}mr^2 \qquad \text{……..(xxii)}$$

By solving these last two equations for $v(t)$ and $\omega(t)$ with initial conditions $v(0)=0$ and $\omega(0)=\frac{R}{r}\Omega$, and imposing the condition $v'(t)=\omega'(t)r$ for the onset of pure rolling we get

$$v' = \frac{1}{3}R\Omega, \quad \omega' = \frac{1}{3}\frac{R}{r}\Omega \qquad \text{……..(xxiii)} \qquad \text{(0.8 point)}$$

From now on m will roll up the side of the cylindrical wall. And since frictional force does not do work in pure rolling we can use the principle of conservation of energy.

$$\frac{1}{2}mv^2+\frac{1}{2}I_{\mathrm{CM}}\omega^2+2mg\left(R-r\right) = \frac{1}{2}mv'^2+\frac{1}{2}I_{\mathrm{CM}}\omega'^2 \quad \text{……..(xxiv)} \quad \text{(0.8 point)}$$

We have also $$N = m\frac{v^2}{R-r}-mg \quad \text{……..(xxv)} \quad \text{(0.2 point)}$$

$\therefore$ $$N = \left(\frac{m}{R-r}\right)\left(\frac{R\Omega}{3}\right)^2-\frac{11}{3}mg \quad \text{……..(xxvi)} \quad \text{(0.2 point)}$$

m will reach the top if $N \geq 0$.

Hence $$\Omega \geq \sqrt{33g\left(\frac{R-r}{R^2}\right)} \quad \text{(0.3 point)}$$

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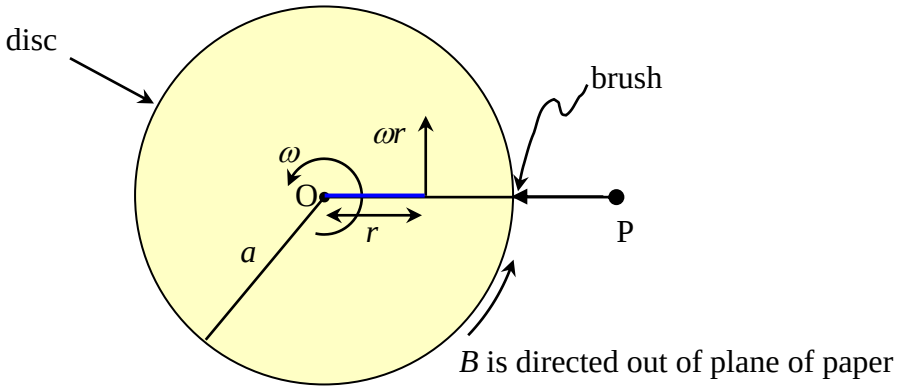
A Self-excited Magnetic Dynamo

2.1) $$L\frac{d}{dt}i + Ri = \mathrm{E} \qquad \text{……(i)}$$ (1.0 point)

2.2) If the length ℓ of solenoid is much greater than its diameter, then the magnetic flux density in the inside mid-section of the solenoid is given by

$$B = \frac{\mu_0 N i}{\ell} \qquad \text{…..……(ii)}$$ (1.5 points)

2.3)



The electric field intensity (E) at distance r from the centre of the axle is

$$E = B\omega r$$

pointing towards the rim of the disc. The induced e.m.f. (E) between terminals P and Q is given by

$$\mathrm{E} = \int_{r=0}^{a} B\omega r dr = \frac{1}{2}B\omega a^2 = \frac{\mu_0 N i \omega a^2}{2\ell} \qquad \text{……(iii)}$$ (2.0 points)

2.4) By combining the results in 2.1) and 2.3) we get

$$L\frac{d}{dt}i + Ri = \left(\frac{\mu_0 N a^2 \omega}{2\ell}\right) i$$

$$\frac{d}{dt}i = +\frac{1}{L}\left(\frac{\mu_0 N a^2 \omega}{2\ell} - R\right) i \equiv +\gamma i$$ (0.5 point)

$$\therefore \quad i(t) = i(0)e^{\gamma t} \qquad \text{……(iv)}$$ (1.0 point)

where $\gamma \equiv \frac{1}{L}\left(\frac{\mu_0 N a^2 \omega}{2\ell} - R\right)$

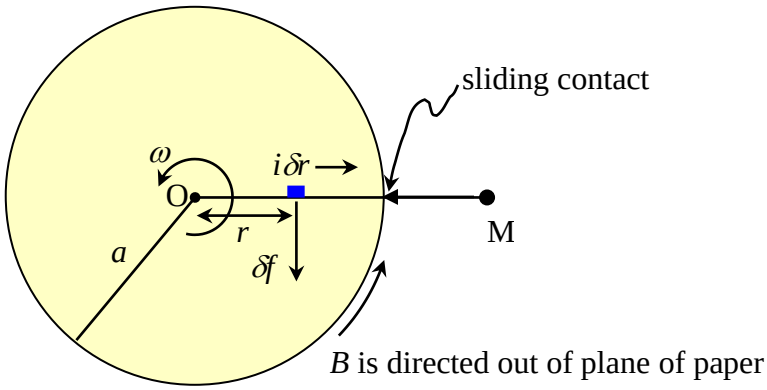
2.5) In order that the current $i(t)$ will grow, the value of γ must be positive otherwise the current will gradually decay.

$$\frac{\mu_0 N a^2 \omega}{2\ell} - R \geq 0 \qquad \text{(1.5 points)}$$

$$\omega_{\min} = \frac{2\ell R}{\mu_0 N a^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\text{(v)} \qquad \text{(0.5 point)}$$

2.6)

Method 1



At the instant t the current is given in 4) as $i(t) = i(0)e^{\gamma t}$.

The magnetic force δf on the current element $i\delta r$ is $\delta f = Bi\delta r$. (1.0 point)

The torque $\delta\tau = r\delta f = Bir\delta r$ opposes the rotation of the disc.

The total torque $\tau = \int_{r=0}^{r=a} Bir\delta r = \frac{1}{2}Bia^2$

$$\tau = \frac{\mu_0 N a^2}{2\ell} i^2 = \frac{\mu_0 N a^2}{2\ell} i^2(0) e^{+2\gamma t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\text{(vi)} \qquad \text{(1.0 point)}$$

In order to maintain the angular velocity of the disc at a steady value we must apply a turning torque of equal magnitude and of opposite direction to that of (vi).

Method 2

$$\tau\omega = i^2R + \frac{d}{dt}\left(\text{magnetic energy in solenoid}\right) \qquad \text{(1.0 point)}$$

$$= i^2R + L\frac{di}{dt}i$$

$$\tau = \frac{\mu_0 Na^2}{2\ell}i^2 \quad = \quad \frac{\mu_0 Na^2}{2\ell}i^2(0)e^{+2\gamma t} \qquad \text{(1.0 point)}$$

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The Leidenfrost Phenomenon

3.1) As given in the problem

$$\frac{d}{dz}v = \left(\frac{1}{\eta}\frac{dP}{dr}\right)\cdot z \qquad \ldots\ldots\ldots\ldots \text{(i)}$$

Integrating (i) with respect to z, we get

$$v(z) = \left(\frac{1}{2\eta}\frac{dP}{dr}\right)\cdot z^2 + C \qquad \ldots\ldots\ldots\ldots \text{(ii)} \qquad \text{(0.5 point)}$$

3.2)
$$v\left(\frac{b}{2}\right) = 0 = \left(\frac{1}{2\eta}\frac{dP}{dr}\right)\cdot\left(\frac{b}{2}\right)^2 + C \qquad \ldots\ldots\ldots\ldots \text{(iii)}$$

$$\therefore \qquad C = -\frac{b^2}{8\eta}\frac{dP}{dr} \qquad \text{(0.5 point)}$$

Note that C is not a real constant; its value depends on $\frac{dP}{dr}$ which is a function of r.

3.3) Let Q be the volume rate of flow of the vapour through the cylindrical surface of $2\pi rb$.

$\delta Q = v(z)\cdot 2\pi r\delta z$ where from (ii) and (iii): (0.3 point)

$$v(z) = \left(\frac{1}{2\eta}\frac{dP}{dr}\right)\cdot\left[z^2 - \frac{b^2}{4}\right] \qquad \ldots\ldots\ldots\ldots \text{(iv)}$$

$$\therefore \qquad Q = 2\int_{z=0}^{\frac{b}{2}} v(z)\cdot 2\pi r dz = \left(\frac{2\pi r}{\eta}\frac{dP}{dr}\right)\int_{z=0}^{\frac{b}{2}}\left[z^2 - \frac{b^2}{4}\right]dz$$

$$Q = -\frac{\pi r b^3}{6\eta}\frac{dP}{dr} \qquad \ldots\ldots\ldots\ldots\ldots \text{(v)} \qquad \text{(0.7 point)}$$

3.4) The total rate of heat flow from the area πr^2 of the hot surface to the drop is $\frac{\pi r^2 \mathrm{K}\,\Delta T}{b}$. We assume that this heat goes into vaporizing the drop.

Hence $\rho Q \ell = \frac{\pi r^2 \mathrm{K} \Delta T}{b}$ and using (v) we get

$$\frac{dP}{dr} = -\left(\frac{6\eta \mathrm{K}\Delta T}{\rho_{\mathrm{V}} \ell b^4}\right) \cdot r \qquad \ldots\ldots\ldots\ldots\ldots\ldots \text{(vi)} \qquad \text{(0.4 point)}$$

This gives $P(r) = -\left(\frac{3\eta \mathrm{K}\Delta T}{\rho_{\mathrm{V}} \ell b^4}\right) \cdot r^2 + B$ (0.4 point)

where B is an arbitrary constant whose value can be found by applying the boundary condition $P(R) = P_{\mathrm{a}}$, the atmospheric pressure.

Hence $B = P_{\mathrm{a}} + \left(\frac{3\eta \mathrm{K}\Delta T}{\rho_{\mathrm{V}} \ell b^4}\right) \cdot R^2$ …… (vii) (0.4 point)

and $P(r) = P_{\mathrm{a}} + \left(\frac{3\eta \mathrm{K}\Delta T}{\rho_{\mathrm{V}} \ell b^4}\right) \cdot \left(R^2 - r^2\right)$ …… (viii) (0.8 point)

3.5) The net force due to pressure is in the upward direction and of magnitude

$$f = \int_{r=0}^{R} \left[P(r) - P_a\right] 2\pi r dr = \frac{3\pi\eta \mathrm{K}\Delta T R^4}{2\rho_V \ell b^4} \quad \text{.....(ix)} \qquad \text{(1.0 point)}$$

The weight of the drop is $\frac{2}{3}\pi R^3 \rho_0 g$, where ρ_0 is the density of liquid.

$$\therefore \quad \frac{2}{3}\pi R^3 \rho_0 g = \frac{3\pi\eta \mathrm{K}\Delta T R^4}{2\rho_V \ell b^4}$$

$$b = \left(\frac{9\eta \mathrm{K} R \Delta T}{4\rho_0 \rho_V \ell g}\right)^{\frac{1}{4}} \quad \text{.....(x)}$$

Note that $\frac{3\eta \mathrm{K}\Delta T}{\rho_V \ell b^4} = \frac{4}{3}\frac{\rho_0 g}{R}$ .....(xi) (1.0 point)

3.6) Use equations (xi) and (viii) to obtain

$$P(r) = P_a + \left(\frac{4}{3}\frac{\rho_0 g}{R}\right)\cdot\left(R^2 - r^2\right) \quad \text{.....(xii)}$$

$$\frac{d}{dr}P(r) = -\left(\frac{8}{3}\frac{\rho_0 g}{R}\right)\cdot r \quad \text{.....(xiii)} \qquad \text{(0.8 point)}$$

Then use (v) to calculate the total mass-rate of vaporization $Q\rho_V$ at $r = R$:

$$Q\rho_V = \left(\frac{2\pi b^3 R}{12\eta}\right)\left(\frac{8}{3}\frac{\rho_0 g}{R}\right) R\rho_V = \left(\frac{4\pi\rho_V \rho_0 g R}{9\eta}\right) b^3$$

$$= \left(\frac{4\pi\rho_V \rho_0 g R}{9\eta}\right)\left(\frac{9\eta \mathrm{K} R \Delta T}{4\rho_0 \rho_V \ell g}\right)^{\frac{3}{4}}$$

$$= \left(\frac{4\pi^4 \mathrm{K}^3 \rho_V \rho_0 g (\Delta T)^3}{9\eta \ell^3}\right)^{\frac{1}{4}} \cdot R^{\frac{7}{4}} = \beta R^{\frac{7}{4}} \quad \text{..... (xiv)} \qquad \text{(1.2 points)}$$

3.7) The life-time (τ) of the drop, is to be found from

$$\frac{d}{dt}\left(\frac{2}{3}\pi R^3\rho_0\right) = -Q\rho_V = -\beta R^{\frac{7}{4}}$$

$$R^{\frac{1}{4}}\frac{d}{dt}R = -\frac{\beta}{2\pi\rho_0}$$

$$\int_R^0 R^{\frac{1}{4}}dR = -\int_0^\tau \frac{\beta}{2\pi\rho_0}dt \qquad \text{(1.0 point)}$$

$$\tau = \frac{8\pi\rho_0}{5\beta}R^{\frac{5}{4}} = \frac{8}{5}\left(\frac{9\eta\rho_0^3\ell^3}{4K^3\rho_V g\left(\Delta T\right)^3}\right)^{\frac{1}{4}}\cdot R^{\frac{5}{4}} \qquad \text{(1.0 point)}$$

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Problem 1: The Earth's Horizontal Magnetic Field

Section I

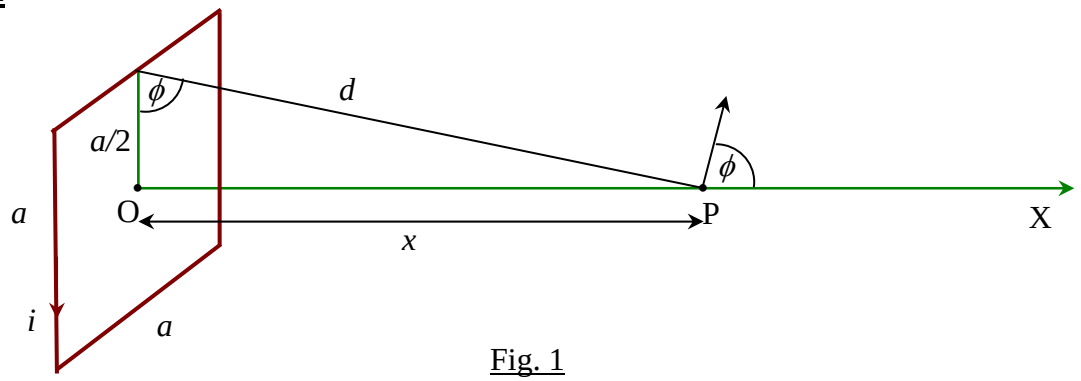


Fig. 1

At O, the centre of coil, the magnetic field for a single turn is

$$B_{\mathrm{O}} = 4\times\frac{\mu_0 i}{2\pi\left(\frac{a}{2}\right)}\frac{(a/2)}{\sqrt{\left(\frac{a}{2}\right)^2+\left(\frac{a}{2}\right)^2}} = \frac{2\sqrt{2}\mu_0 i}{\pi a}.$$

At P, the horizontal magnetic field is

$$B_{\mathrm{PX}} = 4\frac{\mu_0 i}{2\pi d}\frac{(a/2)}{\sqrt{d^2+\left(\frac{a}{2}\right)^2}}\cos\phi\,. \qquad \text{[0.3 point]}$$

From Fig. 1 we have $d = \sqrt{x^2+\left(\frac{a}{2}\right)^2}$ and $\cos\phi = \frac{(a/2)}{\sqrt{x^2+\left(\frac{a}{2}\right)^2}}$. [0.2 point]

Then, for a square coil of N turns [0.2 point]

$$B_{px} = \left(\frac{2\mu_0 iN}{\pi}\right)\cdot\frac{a/2}{\sqrt{x^2+2\left(\frac{a}{2}\right)^2}}\cdot\frac{a/2}{\left(x^2+\left(\frac{a}{2}\right)^2\right)}$$

or

$$B_{px} = \left(\frac{\mu_0 a^2 iN}{2\pi}\right)\left[\frac{1}{\left(x^2+\left(\frac{a}{2}\right)^2\right)\sqrt{x^2+2\left(\frac{a}{2}\right)^2}}\right] \qquad \text{[0.3 point]}$$

which becomes $B_O = \frac{2\sqrt{2}\mu_0 iN}{\pi a}$ as $x = 0$.

Section II

Measurements to justify that we can ignore the torsion of the string**.**

length of string (cm)	time for 10 oscillations (sec)
2	9.38
4	9.69
6	9.90
8	10.13
10	10.13
12	10.22
14	10.12
25	10.12

(Note that this data is from a different magnet used in Section III.)

We can see that the period is constant for length of string ≥ 10 cm.

Section III

The distance between the center of the magnet and the top surface of the platform for Part a), b) and c) is 14.0 ± 0.5 cm.

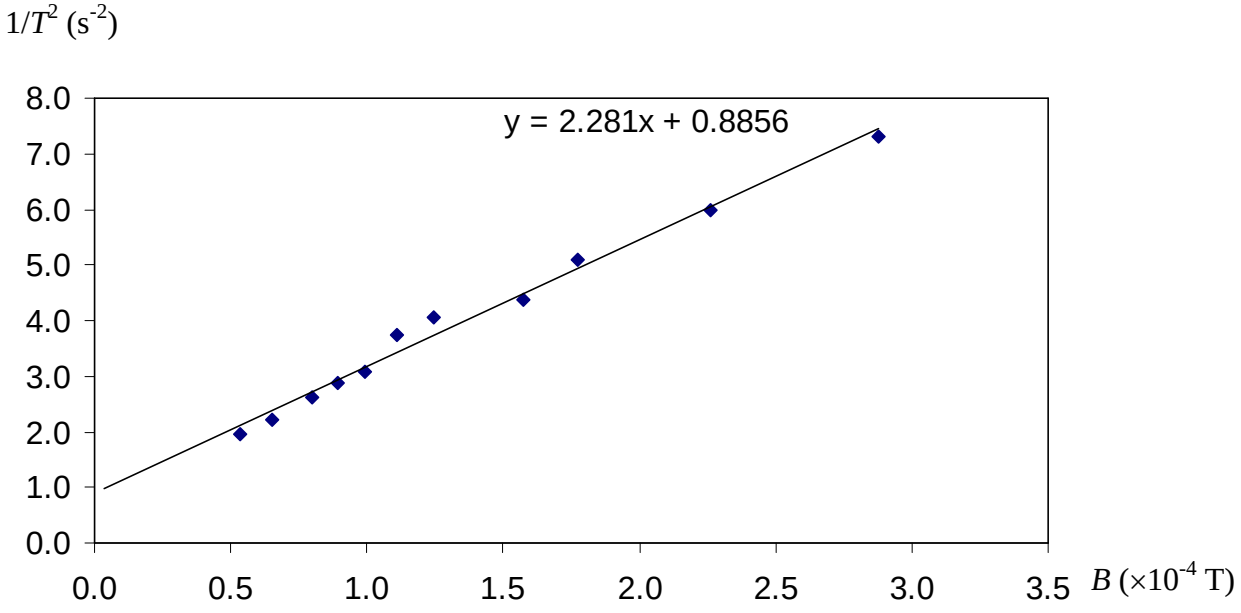
a) Coil's magnetic field and Earth's horizontal magnetic field are in the same direction

Since the coil's magnetic field (B) and Earth's magnetic field (B_H) are in the same direction, from

$T = 2\pi\sqrt{\frac{I}{mB}}$ we have $\frac{1}{T^2} = \beta B + \beta B_H$ where $\beta = \frac{m}{4\pi^2 I}$ By plotting linear graph of $\frac{1}{T^2}$ and B we can find B_H from its slope and intercept.

Measurement of 20 oscillations at different distances from coil, we get the result as in table.

x (cm)	time for 20 oscillation (sec)		period T (sec)	$B\left(\times 10^{-4}\mathrm{T}\right)$	$1/T^2$
10	7.44	7.35	0.370	2.878	7.305
12	8.19	8.13	0.408	2.259	5.998
14	8.87	8.91	0.443	1.773	5.088
15	9.5	9.62	0.478	1.573	4.377
17	9.91	9.97	0.497	1.245	4.048
18	10.43	10.35	0.518	1.111	3.734
19	11.47	11.31	0.569	0.994	3.085
20	11.78	11.81	0.591	0.891	2.866
21	12.41	12.34	0.619	0.801	2.613
23	13.41	13.4	0.671	0.652	2.222
25	14.22	14.28	0.714	0.535	1.964



From graph we have: slope $\beta = (2.281 \pm 0.063) \times 10^4$ $\mathrm{s}^{-2}/\mathrm{T}$

intercept $\beta B_{\mathrm{H}} = 0.886 \pm 0.076$ s^{-2}

The value of Earth's magnetic field is

$$B_{\mathrm{H}} = \frac{0.8856}{2.281 \times 10^4} = 0.39 \times 10^{-4}\ \mathrm{T} = 0.39 \pm 0.04\ \mathrm{G}$$

The magnetic moment of magnet is $m = \beta^2 4\pi^2 M \left(\frac{L^2}{12} + \frac{r^2}{4}\right) = 1.68 \pm 0.09\ \mathrm{A\ m^2}$

b) Earth's magnetic field only

Time for 30 oscillations: 36.28, 36.25, 36.24 s.

Averaged period $T_E = 1.209 \pm 0.001$ s

$$B_{\mathrm{H}} = \frac{1}{T_E^{\,2}\beta} = \frac{1}{1.21^2 \times 2.281 \times 10^{-4}} = 0.30 \pm 0.01\,\mathrm{G}$$

c) Coil's magnetic field and Earth's horizontal magnetic field are in opposite directions

The equilibrium (neutral) position $x_0 = 31.0 \pm 0.2$ cm.

$$B_{\mathrm{H}} = \left(\frac{\mu_0 a^2 iN}{2\pi}\right)\left[\frac{1}{\left(x_0^{\,2} + \left(\frac{a}{2}\right)^2\right)\sqrt{x_0^{\,2} + 2\left(\frac{a}{2}\right)^2}}\right] = 0.31 \pm 0.01\,\mathrm{G}$$

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Problem2: Oscillation of Water-Filled Vessel

Section I

i) $m_1 = \rho\pi\left[R^2-(R-t)^2\right]L = \rho\pi\left(2Rt-t^2\right)L$ …….. (i)

ii) $m_2 = \rho\pi(0.6\,\text{cm})R^2$ …….. (ii)

iii) $m_3 = \pi(R-t)^2 L$ …….. (iii)

iv) $M = m_1+2m_2+m_3$ …….. (iv)

v) Water, as an **ideal** fluid, does not take part in the oscillatory motion of the water-filled vessel. We therefore shall not include contribution of water in the expressions for the moments of inertia.

$$I_y = \frac{1}{2}m_1\left[R^2+(R-t)^2\right]+2\left[\frac{1}{2}m_2R^2\right] \quad \text{…….. (v)}$$

$\ell = 35.6$ cm, $g = 978$ cm s^{-2}, $a = R = 2.5$ cm,

$\rho = 2.70$ g cm^{-3}, $h = 9.2$ cm, $L = 9.2-1.2 = 8.0$ cm

$m_1 = 339.3t-67.86t^2$ g,

$m_2 = 31.8$ g,

$m_3 = 157.1-125.7t+25.13t^2$ g,

$M = 220.7+213.6t-42.73t^2$ g

$I_y = 198.8+2121t-1273t^2+339.3t^3-33.93t^4$

Section II

a)

$$T_y = 2\pi\sqrt{\frac{\ell}{g}\cdot\frac{I_y}{Ma^2}} \quad \text{…….. (vi)}$$

Time for 50 oscillations is 43.3 s, hence

$T_y = 0.866$ s ± 0.004 s

Substituting these values into equation (vi), we get

$t^4-10t^3+33.42t^2-41.97t+15.36 = 0$ …….. (vii)

The solution of (vii), by numerical iteration, is $t = 0.62$ cm $\pm$ 0.02 cm

(The error can be estimated from a repeat of the procedure or a differential equation for dT and dt)

Hence, we get

$m_1 = 184 \pm 5$ g,

$m_2 = 31.8 \pm 0.2$ g,

$m_3 = 89 \pm 2$ g,

$M = 337 \pm 6$ g.

b)

$$T_x = 2\pi\sqrt{\frac{\ell}{g}\cdot\frac{I_x}{Ma^2}} \qquad \text{..... (viii)}$$

$\ell = 33.6$ cm, $a = \frac{h}{2} = \frac{9.2}{2} = 4.6$ cm

Time for 50 oscillations is 38.0 s $\pm$ 0.2 s, hence

$T_x = 0.760$ s $\pm$ 0.004 s

And from $T_x = 2\pi\sqrt{\frac{\ell}{g}\cdot\frac{I_x}{Ma^2}}$ where $\ell = 33.6$ cm, $a = \frac{h}{2} = \frac{9.2}{2} = 4.6$ cm,

$g = 978$ cm s^{-2}, $M = 337$ g , we get

$I_x^{\text{Exp}} = 3036 \text{ g cm}^2 \pm 94 \text{ g cm}^2$

Also $I_x^{\text{Theo}} = 3261 \text{ g cm}^2 \pm 68 \text{ g cm}^2$

c)

$\Delta I_x = I_x^{\text{Theo}} - I_x^{\text{Exp}} = 225$ g cm^2

The experimental value I_x^{Exp} is smaller than I_x^{Theo} by 225 g cm^2.

This difference is probably significant and it is due to low viscosity of water. The mass of water in the middle section does not take part in the oscillatory motion of the vessel.

ΔI_x can be estimated to be due to a stationary cylindrical portion of water in the middle.

$$\Delta I_x = \pi (R-t)^2 L_{water} \left(\frac{L_{water}^{\ 2}}{12} + \frac{(R-t)^2}{4} \right)$$

$L_{water} \approx 5.7\,\text{cm}$

This corresponds to the water mass of $\approx 63.8\,g$

The percentage of the water that takes part in the oscillation is $\approx 28.5\%$

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10th APhO

Experimental Competition

28 April 2009

Marking Scheme

General marking guidelines

1. Giving correct answers without calculations	Deduct up to 50% of the total points for that part
2. Minor mistakes in the calculations e.g. wrong signs, symbols, substitutions	Deduct 30%
3. Missing units in the final numerical answers (for each part)	Deduct 0.1 point
4. Final answers (for each part) containing too few or too many significant figures.	Deduct 0.1 point
5. Using wrong physical concepts (despite correct answers)	No points awarded
6. Error propagated from earlier parts: minor errors	Full points
7. Error propagated from earlier parts: major errors (such that the solution becomes trivial).	Deduct up to 20%

Marking Scheme: Problem 1

No.	Description	Scores
Section I	Derivation of formula	**1 point**
	- the horizontal magnetic field for a coil of a single turn $B_{PX} = 4\frac{\mu_0 i}{2\pi d}\frac{(a/2)}{\sqrt{d^2+\left(\frac{a}{2}\right)^2}}\cos\phi$ (0.3 point) $d = \sqrt{x^2+\left(\frac{a}{2}\right)^2}$ (0.1 point) $\cos\phi = \frac{(a/2)}{\sqrt{x^2+\left(\frac{a}{2}\right)^2}}$ (0.1 point) - For a coil of *N* turns, replace the current with iN (0.2 point) $B_{px} = \left(\frac{\mu_0 a^2 iN}{2\pi}\right)\left[\frac{1}{\left(x^2+\left(\frac{a}{2}\right)^2\right)\sqrt{x^2+2\left(\frac{a}{2}\right)^2}}\right]$ (0.3 point)	
Section II		
	Measurements to justify that we can ignore the torsion of the string	**0.8 point**
	- Tabulated data of lengths and periods (total 0.8 point) - At least 1 data point for length of the string from 0 to 10 cm (where the effect of torsion is prominent (0.2 point) - At least 3 data points for length of the string greater than 10 cm (where the effect of torsion is negligible) (0.2 each = 0.6 point)	

Section III		
	- **Provide the value of the distance**	**0.2 point**
(a)	**Coil's magnetic field and Earth's magnetic field are in the same direction**	**5.0 points**
	- Show that $\frac{1}{T^2} = \beta B + \beta B_{\text{H}}$, $\beta = \frac{m}{4\pi^2 I}$ (0.2 point) - Tabulated data from measurements for at least 5 different values of x (total 3.0 points) Provide values of B (1.0 point) Provide values of T (minimum 5 measurements with at least 10 oscillations each) (1.5 points) Provide values of $\frac{1}{T^2}$ (0.5 point) - Provide a complete graph of $\frac{1}{T^2}$ and B (0.8 point) - Provide a value of the slope of the graph. (0.3 point) - Provide a value of the interception of the graph (0.1 point) - Provide a numerical value of B_{H} (0.2 point) - Provide a numerical value of m (0.2 point) - Error estimation (0.2 point)	
(b)	**Earth's magnetic field only**	**1.0 point**
	Measure the period accurately (minimum 5 measurements with at least 10 oscillations each) (0.3 point) B_{H} is from 0.25 to 0.35 gauss (max 0.5 point) B_{H} is 0.1- 0.25 or 0.4 - 0.5 gauss (0.3 point) Otherwise (0.0 point) - Error estimation (0.2 point)	

(c)	**Coil's magnetic field and Earth's magnetic field are in opposite direction**	**2.0 points**
	If B_H is from 0.2 to 0.4 gauss, (max. 2.0 points) - Provide the correct value of x_0 and its measurement details. (1.0 point) - Provide the correct value of B_H. (0.5 point) - Estimate the error of B_H. (0.5 point) If B_H is 0.1-0.2 or 0.4-0.5 gauss, (max. 1.0 point) - Provide the correct value of x_0 and its measurement details. (0.3 point) - Provide the correct value of B_H. (0.2 point) - Estimate the error of B_H. (0.5 point) For other value of B_H, - Estimate the error of B_H. (0.5 point)	

********** End of Problem 1**********

Marking Scheme: Problem 2

No.	Description	Scores
Section I	Derivation of formulae	**2.0 points**
	• $m_1 = \rho\pi\left[R^2-\left(R-t\right)^2\right]L = \rho\pi\left(2Rt-t^2\right)L$ (0.1 point) • $m_2 = \rho\pi\left(0.6\,\text{cm}\right)R^2$ (0.1 point) • $m_3 = \pi\left(R-t\right)^2 L$ (0.1 point) • $M = m_1+2m_2+m_3$ (0.1 point) • $I_y = \frac{1}{2}m_1\left[R^2+\left(R-t\right)^2\right]+2\left[\frac{1}{2}m_2R^2\right]$: realize that the water does not contribute to the inertia (0.5 point), correctly use the formula for the disc (0.2 point) • Measurements of R, h, L with errors. (0.3 point) • Numerical expressions: for example $m_1 = 339.3t-67.86t^2$ g, (0.1 point) $m_2 = 31.8$ g, (0.1 point) $m_3 = 157.1-125.7t+25.13t^2$ g, (0.1 points) $M = 220.7+213.6t-42.73t^2$ g (0.1 point) $I_y = 198.8+2121t-1273t^2+339.3t^3-33.93t^4$ g cm^2 (0.2 point)	
Section II		
(a)	**Angular oscillation about the axis of symmetry**	**4.0 points**
	- Provide the measured value of T_y Let n be the number of oscillations. (max. 0.8 point) $n \geq 30$, (0.8 point) ($20 \leq n < 30$, (0.6 point)) ($10 \leq n < 20$, (0.2 point)) ($n < 10$, (0 point))	

	Number of data points (max. 0.8 point) $n \geq 5$, (0.8 point) ($3 \leq n < 5$, (0.6 point)) ($n = 2$, (0.2 point)) ($n < 2$, (0 point)) - Provide the error of T_y (0.2 point) - Provide the polynomial equation for t (0.3 point) - Perform numerical iteration (0.5 point) - Provide the value of t (max. 0.5 point) $0.60 \leq t \leq 0.70$, (0.5 point) $0.50 \leq t < 0.60$ **or** $0.70 < t \leq 0.80$, (0.3 point) $0.80 < t \leq 0.90$, (0.1 point) $t < 0.50$ **or** $t > 0.90$ (0 point) - Provide the numerical values of m_1, m_2, m_3 and M (0.1 each = 0.4 point) - Provide the error of t with a correct method (0.5 point)	
(b)	**Angular oscillation about the central axis perpendicular to the length**	**2.8 points**
	- Provide the measured value of T_x Let n be the number of oscillations. (max. 0.8 point) $n \geq 30$, (0.8 point) ($20 \leq n < 30$, (0.6 point)) ($10 \leq n < 20$, (0.2 point)) ($n < 10$, (0 point)) Number of data points (max. 0.8 point) $n \geq 5$, (0.8 point) ($3 \leq n < 5$, (0.6 point)) ($n = 2$, (0.2 point)) ($n < 2$, (0 point))	

	- Provide the error of T_x (0.2 point) - Provide the numerical value of I_x^{Exp} (0.5 point) - Provide the numerical value of I_x^{Theo} (0.5 point)	
(c)	**Comparing experimental and theoretical values of the moment of inertia**	**1.2 points**
	- Provide the correct value of ΔI_x (0.3 point) - Show that ΔI_x is statistically significant (0.2 point) - Provide the value of the percentage (0.7 point)	

********** End of Problem 2**********